Temperaturverlauf als Funktion von Radius und Zeit im Hohlzylinder

Gerhard Herres, 12.10.2019-22.10.2019

> $restart : Digits := 20 :$

In einem Hohlzylinder mit Innenradius RB=8 m und Aussenradius b=2000 m wird die instationäte Wärmeleitung berechnet.
Die Differentialgleichung führt auf die Grundlösung der Besselfunktionen BesselK(0,c*r) und BesselY (0,c*r),
die im Hohlzylinder beide auftreten können, weil r=0 nicht endlich sein muss.
Nach Carslaw und Jaeger "Heat Conduction in Solids" ist eine Lösung:

> $U := \mathrm{BesselJ}(0, c_k \cdot r) \cdot \mathrm{BesselY}(0, c_k \cdot b) - \mathrm{BesselJ}(0, c_k \cdot b) \cdot \mathrm{BesselY}(0, c_k \cdot r);$
> $Us := diff(U, r);$

$$U := \mathrm{BesselJ}(0, c_k r)\ \mathrm{BesselY}(0, c_k b) - \mathrm{BesselJ}(0, c_k b)\ \mathrm{BesselY}(0, c_k r)$$

$$Us := -\mathrm{BesselJ}(1, c_k r)\ c_k \mathrm{BesselY}(0, c_k b) + \mathrm{BesselJ}(0, c_k b)\ \mathrm{BesselY}(1, c_k r)\ c_k \tag{1}$$

Diese Lösung erfüllt die äussere Randbedingung, U(r=b) =0. Um auch die Randbedingung am inneren Rand U(r=a) =0 zu erfüllen, müssen die Koeffizienten c_k bestimmt werden.

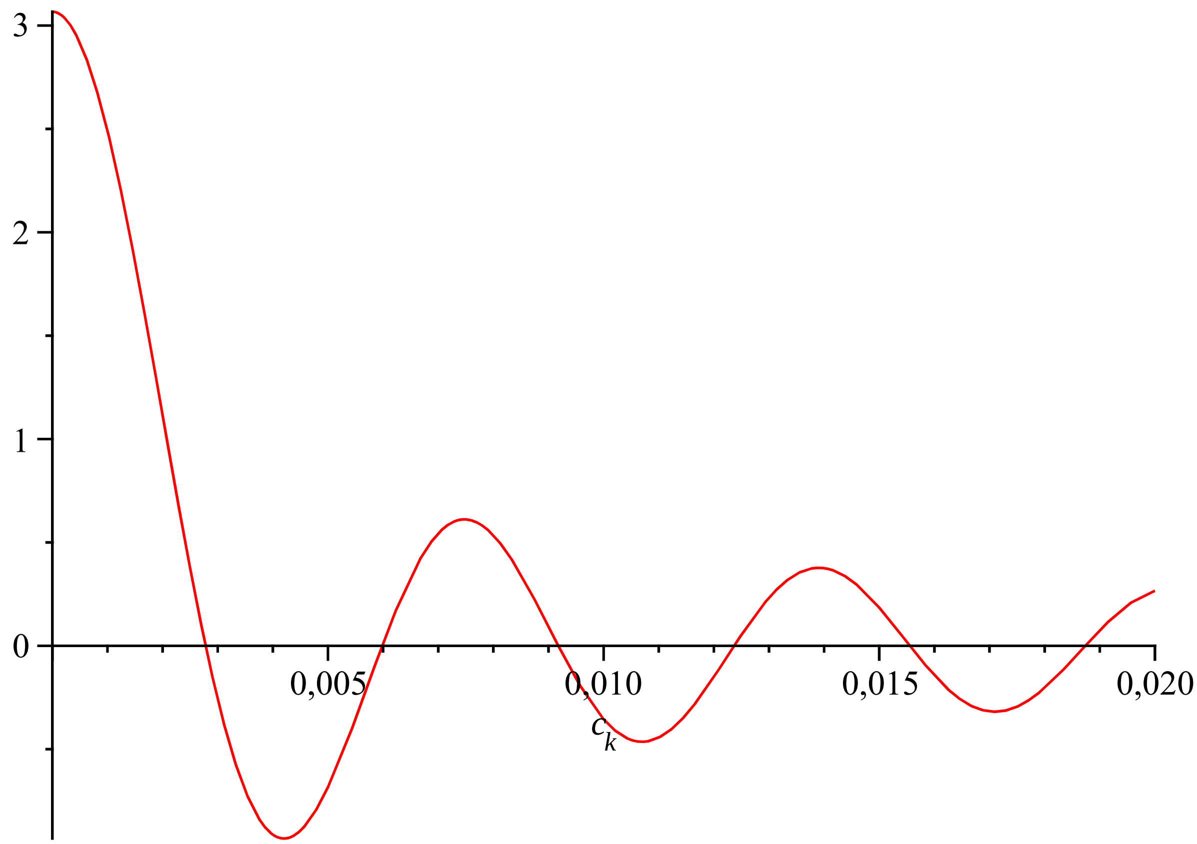
> $a := 8.09 : b := 1000 :$

Randbedingung U(r,a=8.09 m)=0

> $Ua := subs(r = 8.09, U)$

$$Ua := \mathrm{BesselJ}(0, 8.09\ c_k)\ \mathrm{BesselY}(0, 1000\ c_k) - \mathrm{BesselJ}(0, 1000\ c_k)\ \mathrm{BesselY}(0, 8.09\ c_k) \tag{2}$$

> $plot(Ua, c_k = 0.0 ..0.02)$



> $Ts0 := 80 : TL := 25 :$

> $\text{theta} := Ts0 - TL;$

$$\theta := 55 \quad \textbf{(3)}$$

> $c1_1 := fsolve(Ua, c_k = 0.003); c1_2 := fsolve(Ua, c_k = 0.006); dc := c1_2 - c1_1;$

$$c1_1 := 0.0027808276128223511401$$

$$c1_2 := 0.0059817751504845383242$$

$$dc := 0.0032009475376621871841 \quad \textbf{(4)}$$

> $AK_1 := evalf\left(subs\left(c_k = \text{c1}_1, \frac{\pi^2 \cdot c_k^2}{2} \cdot \frac{\text{BesselJ}(0, a \cdot c_k)^2}{\text{BesselJ}(0, a \cdot c_k)^2 - \text{BesselJ}(0, \text{b} \cdot c_k)^2} \cdot int(r \cdot \text{theta} \cdot U, r = \text{a} .. \text{b})\right)\right);$

$BK_1 := evalf\left(subs\left(c_k = \text{c1}_1, \frac{\text{theta} \cdot \pi \cdot \text{BesselJ}(0, a \cdot c_k)}{\text{BesselJ}(0, a \cdot c_k) + \text{BesselJ}(0, b \cdot c_k)}\right)\right);$

$list1 := [[1, c1_1]]; list2 := [[c1_1, AK_1]]; list3 := [[c1_1, BK_1]];$

$$AK_1 := 209.98489150618322568$$

$$BK_1 := 209.98489150618322567$$

$$list1 := [[1, 0.0027808276128223511401]]$$

$$list2 := [[0.0027808276128223511401, 209.98489150618322568]]$$

$$list3 := [[0.0027808276128223511401, 209.98489150618322567]] \quad \textbf{(5)}$$

> $AK_2 := evalf\left(subs\left(c_k = c1_2, \frac{\pi^2 \cdot c_k^2}{2} \cdot \frac{\text{BesselJ}(0, a \cdot c_k)^2}{\text{BesselJ}(0, a \cdot c_k)^2 - \text{BesselJ}(0, b \cdot c_k)^2} \cdot int(r \cdot \text{theta} \cdot U, r = a..b)\right)\right);$

$BK_2 := evalf\left(subs\left(c_k = c1_2, \frac{\text{theta} \cdot \pi \cdot \text{BesselJ}(0, a \cdot c_k)}{\text{BesselJ}(0, a \cdot c_k) + \text{BesselJ}(0, b \cdot c_k)}\right)\right);$

$list1 := [op(list1), [2, c1_2]]; list2 := [op(list2), [c1_2, AK_2]]; list3 := [op(list3), [c1_2, BK_2]];$

$AK_2 := 150.81984650335777212$

$BK_2 := 150.81984650335777212$

list1 := [[1, 0.0027808276128223511401], [2, 0.0059817751504845383242]]

list2 := [[0.0027808276128223511401, 209.98489150618322568],
[0.0059817751504845383242, 150.81984650335777212]]

list3 := [[0.0027808276128223511401, 209.98489150618322567],
[0.0059817751504845383242, 150.81984650335777212]] **(6)**

>

> **for** *l* **from** 3 **to** 300 **do**

$c1_l := fsolve(Ua, c_k = c1_{l-1} + dc):$

$list1 := [op(list1), [l, c1_l]]:$

$\#AK_l := evalf\left(subs\left(c_k = c1_l, \frac{\pi^2 \cdot c_k^2}{2} \cdot \frac{\text{BesselJ}(0, a \cdot c_k)^2}{\text{BesselJ}(0, a \cdot c_k)^2 - \text{BesselJ}(0, b \cdot c_k)^2} \cdot int(r \cdot \text{theta} \cdot U, r = a..b)\right)\right):$

$BK_l := evalf\left(subs\left(c_k = c1_l, \frac{\text{theta} \cdot \pi \cdot \text{BesselJ}(0, a \cdot c_k)}{\text{BesselJ}(0, a \cdot c_k) + \text{BesselJ}(0, b \cdot c_k)}\right)\right);$

$list2 := [op(list2), [c1_l, AK_l]] : list3 := [op(list3), [c1_l, BK_l]];$

end do:

> *list1*

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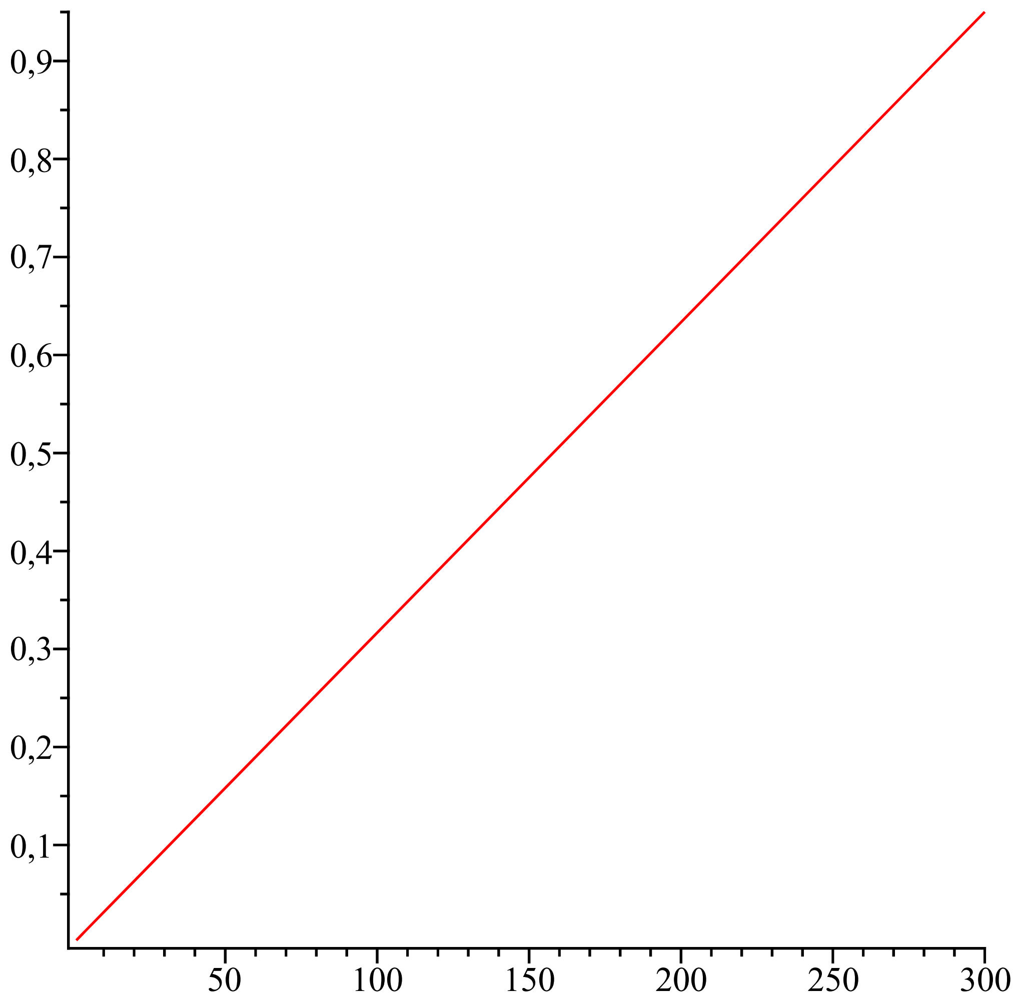
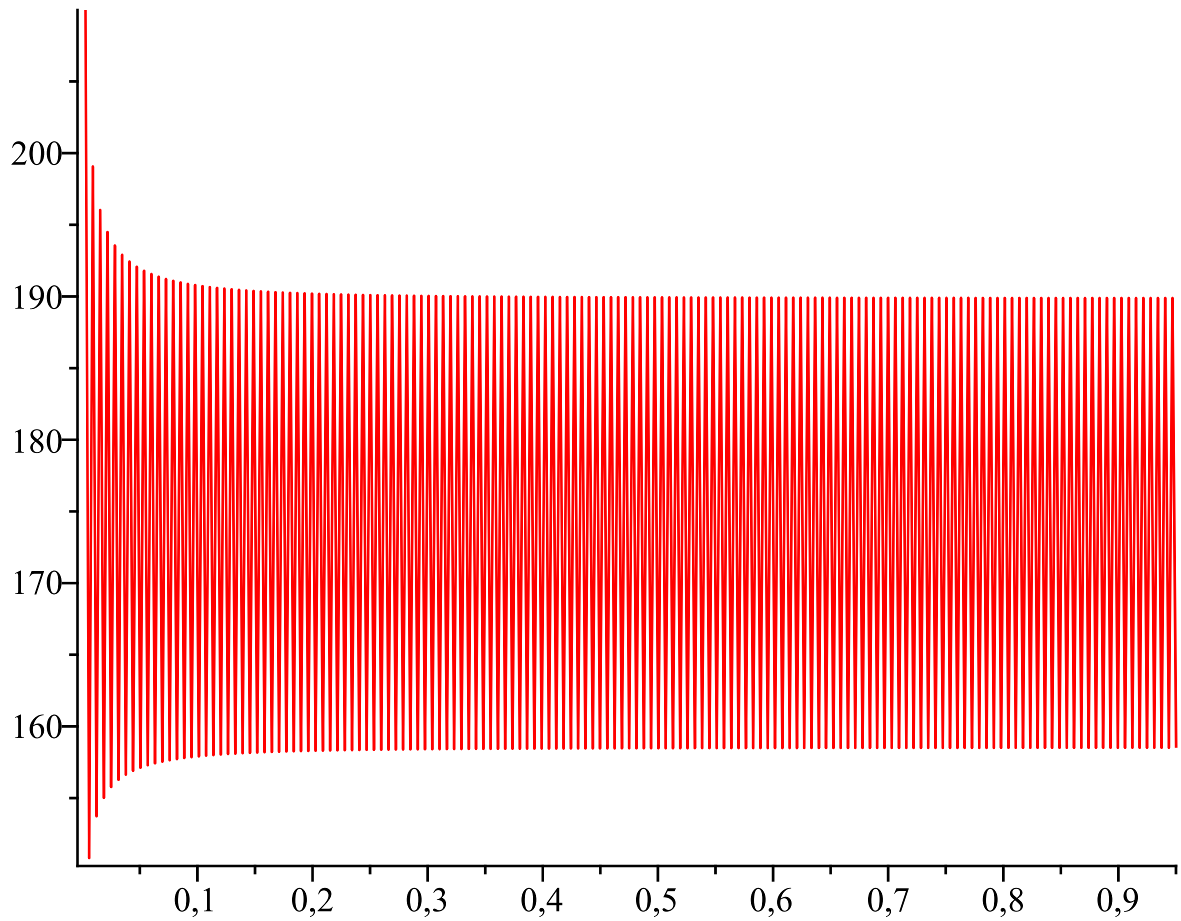
> *plot*(*list1*)



> *plot*(*list3*);



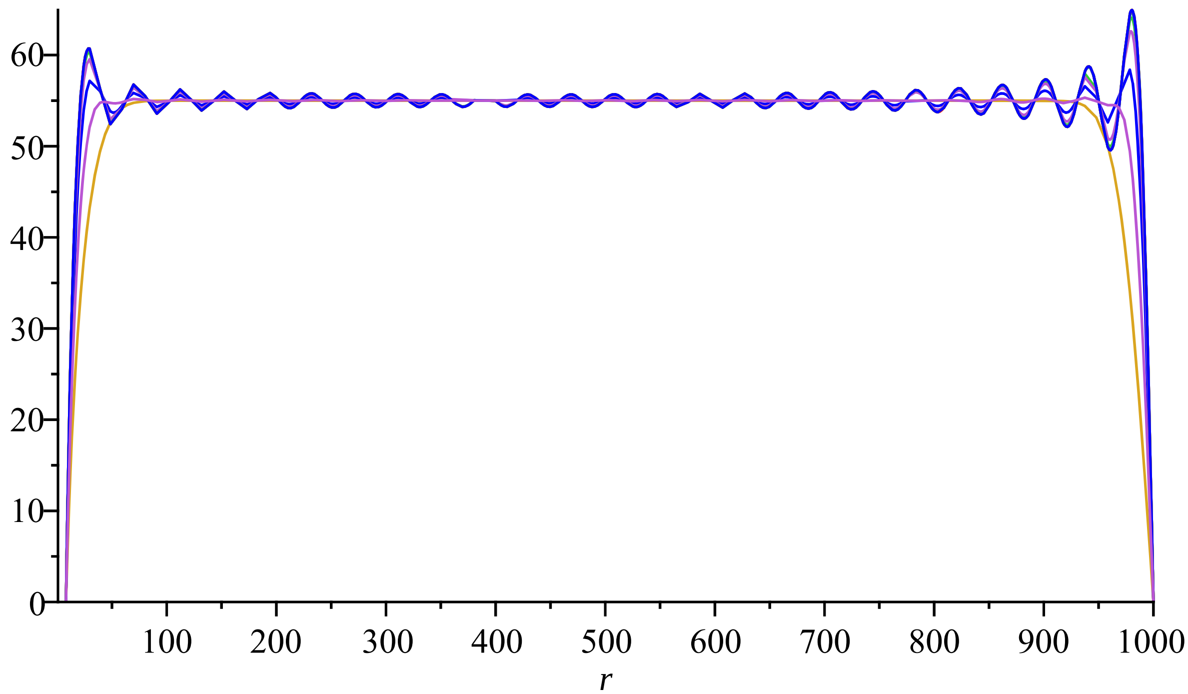
>

> rho := 2100; *cp* := 800; lambda := 5; *at* := $\frac{\lambda}{\rho \cdot cp}$;

$$\rho := 2100$$
$$cp := 800$$
$$\lambda := 5$$
$$at := \frac{1}{336000} \quad \textbf{(8)}$$

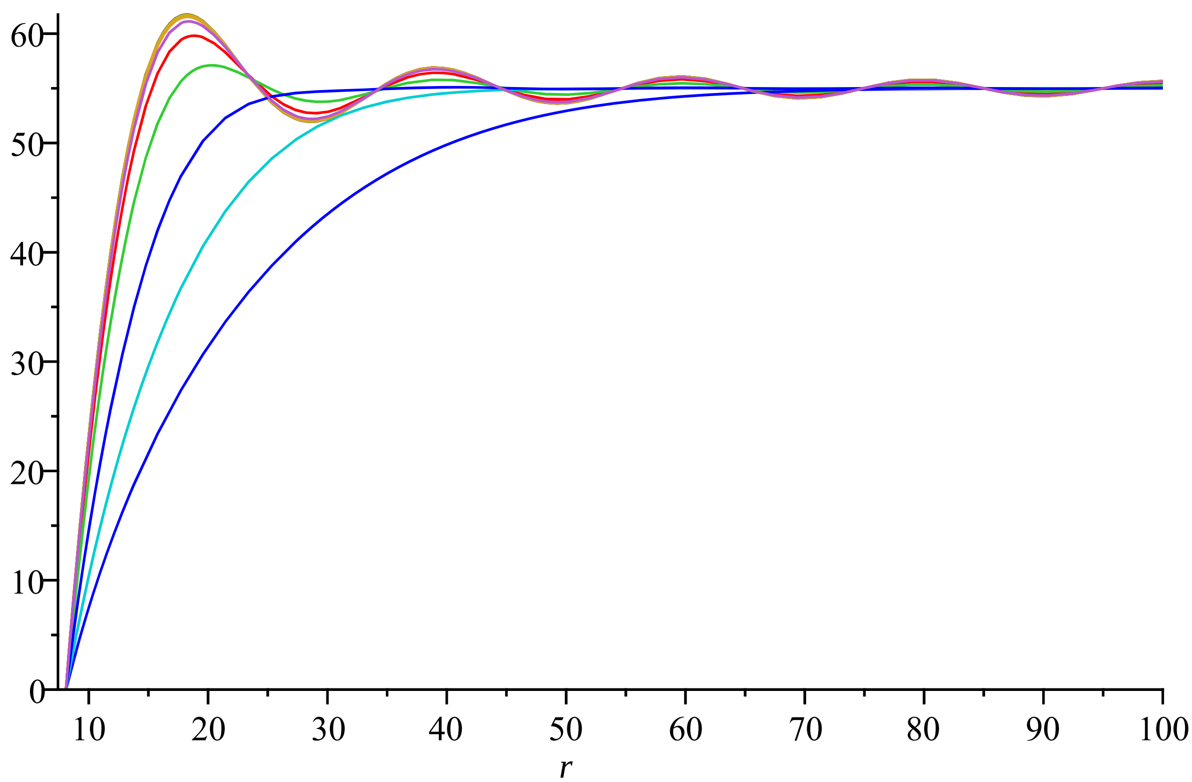
> $T := sum\left(BK_m \cdot subs\left(c_k = c1_m, U\left(c_k \cdot r\right)\right) \cdot \exp\left(-at \cdot c1_m^2 \cdot t\right), m = 1..50\right)$:

> $plot(\{subs(t=10^3, T), subs(t=3\cdot10^3, T), subs(t=10^4, T), subs(t=3\cdot10^4, T), subs(t=10^5, T), subs(t=3\cdot10^5, T), subs(t=10^6, T), subs(t=3\cdot10^6, T), subs(t=10^7, T), subs(t=3\cdot10^7, T), subs(t=10^8, T)\}, r=a..b)$



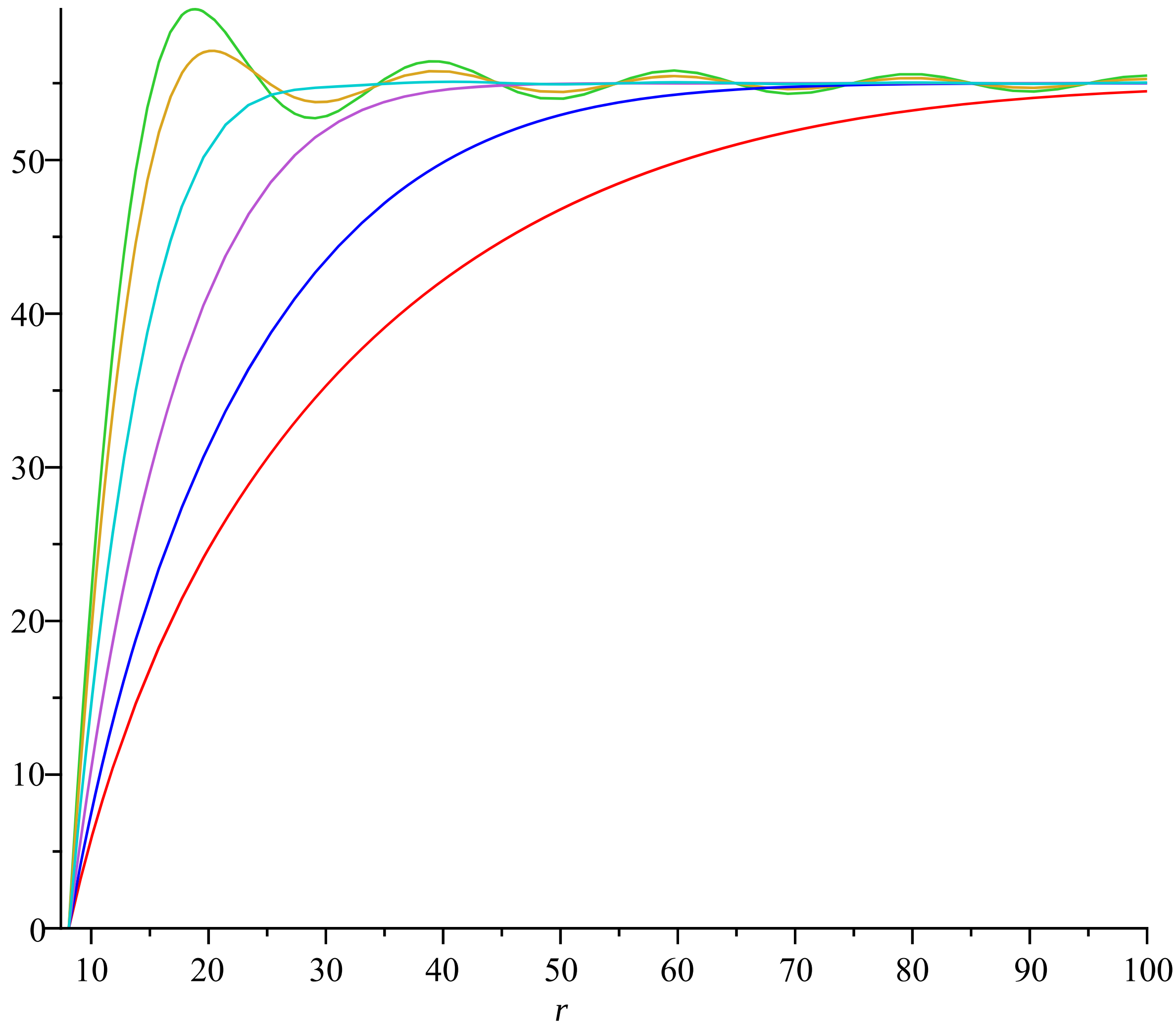
> $T := sum\left(BK_m \cdot subs\left(c_k = c1_m, U\left(c_k \cdot r\right)\right) \cdot \exp\left(-at \cdot c1_m^2 \cdot t\right), m = 1..100\right):$

> $plot(\{subs(t=10^3, T), subs(t=3\cdot 10^3, T), subs(t=10^4, T), subs(t=3\cdot 10^4, T), subs(t=10^5, T), subs(t=3\cdot 10^5, T), subs(t=10^6, T), subs(t=3\cdot 10^6, T), subs(t=10^7, T), subs(t=3\cdot 10^7, T), subs(t=10^8, T)\}, r=a..100);$



> $T := sum\left(BK_m \cdot subs\left(c_k = c1_m, U\left(c_k \cdot r\right)\right) \cdot \exp\left(-at \cdot c1_m^2 \cdot t\right), m = 1..300\right):$

> $plot(\{subs(t=10^6, T), subs(t=3\cdot10^6, T), subs(t=10^7, T), subs(t=3\cdot10^7, T), subs(t=10^8, T), subs(t=3\cdot10^8, T)\}, r=a..100);$



> **for** l **from** 301 **to** 1000 **do**

$c1_l := fsolve(Ua, c_k = c1_{l-1} + dc)$:

$list1 := [op(list1), [l, c1_l]]$:

$\#AK_l := evalf\left(subs\left(c_k = c1_l, \frac{\pi^2 \cdot c_k^2}{2} \cdot \frac{BesselJ(0, a\cdot c_k)^2}{BesselJ(0, a\cdot c_k)^2 - BesselJ(0, b\cdot c_k)^2} \cdot int(r\cdot theta\cdot U, r = a..b)\right)\right)$:

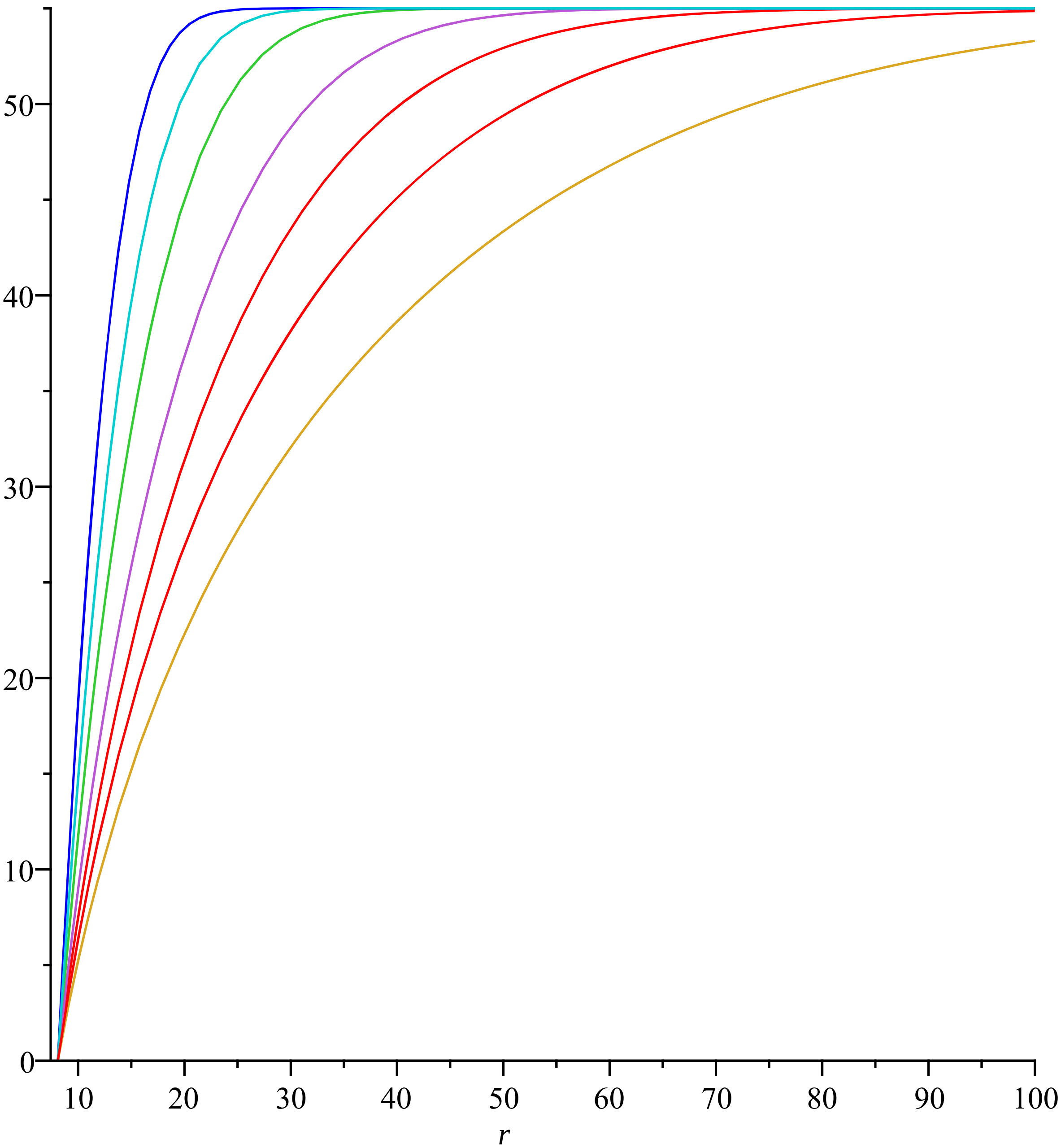
$BK_l := evalf\left(subs\left(c_k = c1_l, \frac{theta\cdot\pi\cdot BesselJ(0, a\cdot c_k)}{BesselJ(0, a\cdot c_k) + BesselJ(0, b\cdot c_k)}\right)\right)$;

$list2 := [op(list2), [c1_l, AK_l]]$: $list3 := [op(list3), [c1_l, BK_l]]$;

end do:

> $T := sum(BK_m \cdot subs(c_k = c1_m, U(c_k\cdot r)) \cdot \exp(-at\cdot c1_m^2 \cdot t), m = 1..1000)$:

> $plot(\{subs(t=5\cdot 10^6, T), subs(t=10^7, T), subs(t=2\cdot 10^7, T), subs(t=5\cdot 10^7, T), subs(t=10^8, T), subs(t=2\cdot 10^8, T), subs(t=5\cdot 10^8, T)\}, r=a..100);$

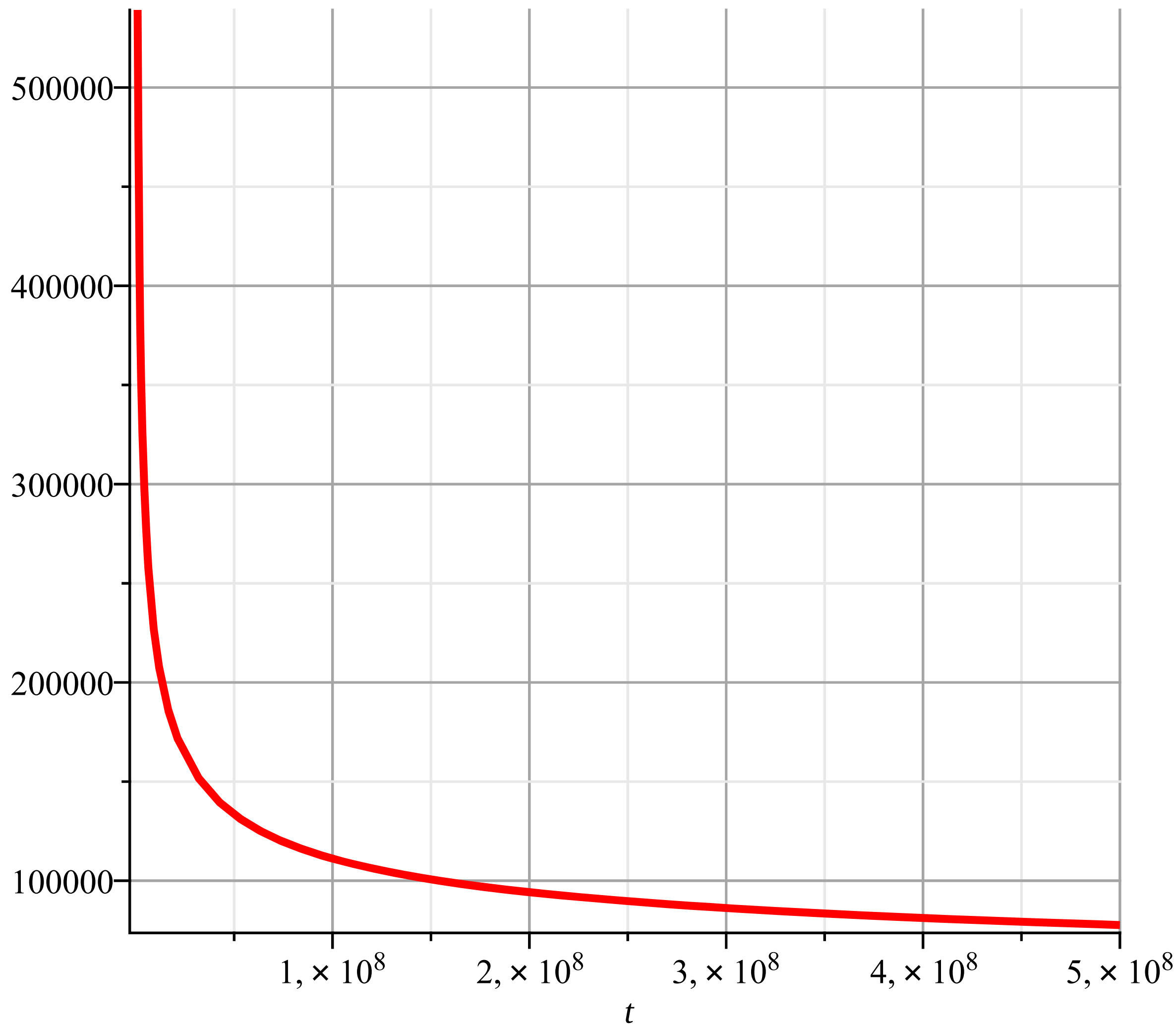


Der Wärmestrom ergibt sich aus der Ableitung: Q=2*π*a*L*λ*diff(T,r)_r=a

> $dT := sum\left(BK_m \cdot subs\left(c_k = c1_m, Us\left(c_k \cdot r\right)\right) \cdot \exp\left(-at \cdot c1_m^2 \cdot t\right), m=1..1000\right):$

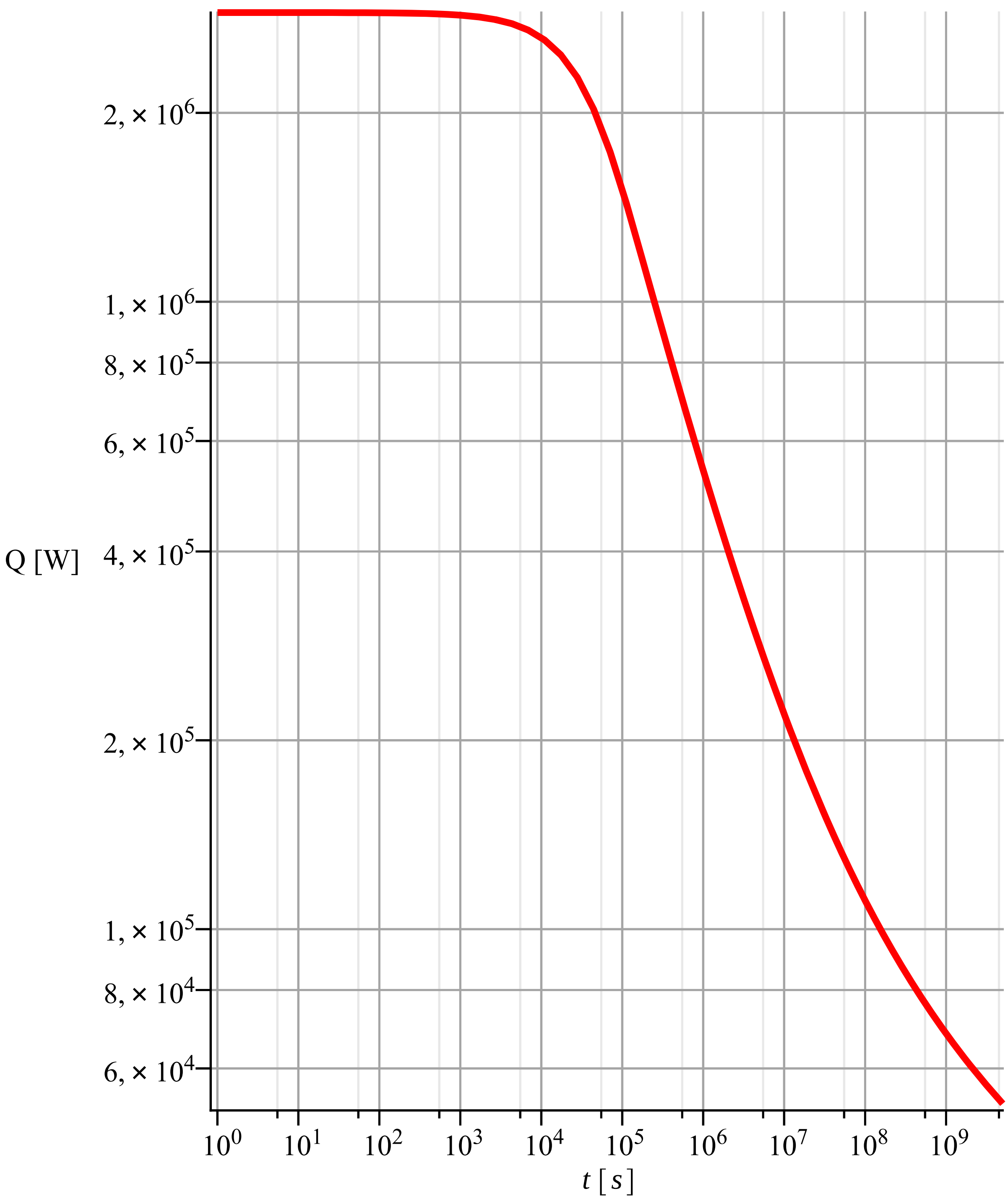
> $L := 100: \; Q := 2\cdot\pi\cdot a\cdot L\cdot \text{lambda}\cdot subs(r=a, dT):$

> $plot(evalf(Q), t=1e6..5e8);$

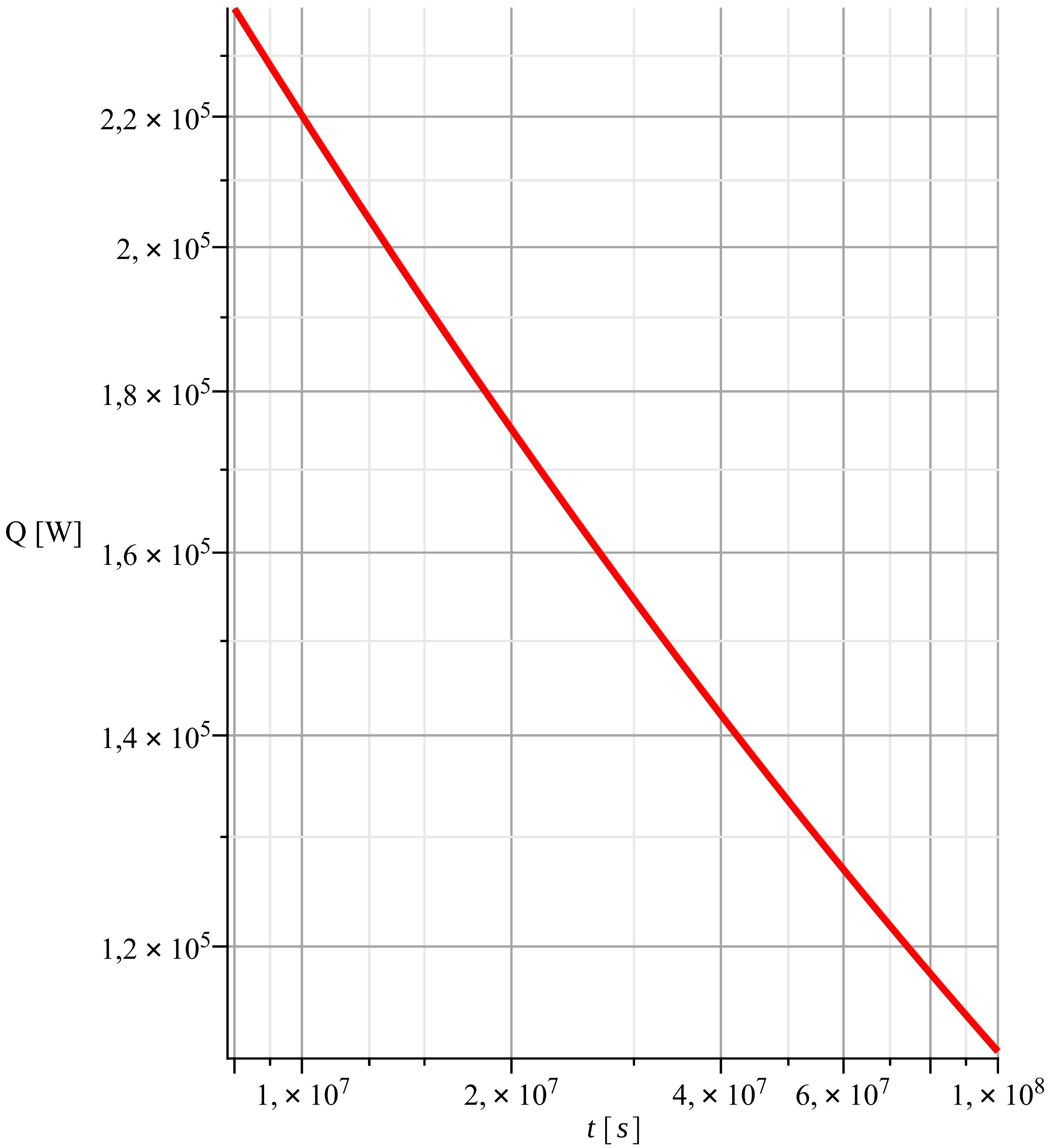


> *with*(*plots*) :

> *loglogplot*(*evalf*(*Q*), *t* = 1e0 ..5e9);



> *loglogplot*(*evalf*(*Q*), *t* = 8e6 ..1e8);



> $plot(\{subs(t=2\cdot 10^3, T), subs(t=5\cdot 10^3, T), subs(t=10^4, T), subs(t=2\cdot 10^4, T), subs(t=5\cdot 10^4, T), subs(t=10^5, T), subs(t=2\cdot 10^5, T), subs(t=5\cdot 10^5, T), subs(t=1\cdot 10^6, T), subs(t=2\cdot 10^6, T), subs(t=5\cdot 10^6, T), subs(t=10^7, T), subs(t=2\cdot 10^7, T), subs(t=5\cdot 10^7, T), subs(t=10^8, T), subs(t=2\cdot 10^8, T), subs(t=5\cdot 10^8, T), subs(t=1\cdot 10^9, T), subs(t=2\cdot 10^9, T)\}, r=a..50);$

> $list4 := [\ ]:$
$subs(r=a+0.1, t=5, T\cdot 10) : evalf(\%); list4 := [op(list4), [5, \%]]:$
$subs(r=a+0.1, t=7, T\cdot 10) : evalf(\%); list4 := [op(list4), [7, \%]]:$

```
subs(r = a + 0.1, t = 10, T·10) :   evalf(%); list4 ≔ [op(list4), [1e1, %]] :
subs(r = a + 0.1, t = 14, T·10) :   evalf(%); list4 ≔ [op(list4), [1.4e1, %]] :
subs(r = a + 0.1, t = 20, T·10) : evalf(%); list4 ≔ [op(list4), [2e1, %]] :
subs(r = a + 0.1, t = 30, T·10) : evalf(%); list4 ≔ [op(list4), [3e1, %]] :
subs(r = a + 0.1, t = 50, T·10) : evalf(%); list4 ≔ [op(list4), [5e1, %]] :
subs(r = a + 0.1, t = 70, T·10) : evalf(%); list4 ≔ [op(list4), [7e1, %]] :
subs(r = a + 0.1, t = 10^2, T·10) :   evalf(%); list4 ≔ [op(list4), [1e2, %]] :
subs(r = a + 0.1, t = 1.4e2, T·10) :   evalf(%); list4 ≔ [op(list4), [1.4e2, %]] :
subs(r = a + 0.1, t = 2·10^2, T·10) : evalf(%); list4 ≔ [op(list4), [2e2, %]] :
subs(r = a + 0.1, t = 3·10^2, T·10) : evalf(%); list4 ≔ [op(list4), [3e2, %]] :
subs(r = a + 0.1, t = 5·10^2, T·10) : evalf(%); list4 ≔ [op(list4), [5e2, %]] :
subs(r = a + 0.1, t = 7·10^2, T·10) : evalf(%); list4 ≔ [op(list4), [7e2, %]] :
subs(r = a + 0.1, t = 10^3, T·10) :   evalf(%); list4 ≔ [op(list4), [1e3, %]] :
subs(r = a + 0.1, t = 1.4e3, T·10) :   evalf(%); list4 ≔ [op(list4), [1.4e3, %]] :
subs(r = a + 0.1, t = 2·10^3, T·10) : evalf(%); list4 ≔ [op(list4), [2e3, %]] :
subs(r = a + 0.1, t = 3·10^3, T·10) : evalf(%); list4 ≔ [op(list4), [3e3, %]] :
subs(r = a + 0.1, t = 5·10^3, T·10) : evalf(%); list4 ≔ [op(list4), [5e3, %]] :
subs(r = a + 0.1, t = 7·10^3, T·10) : evalf(%); list4 ≔ [op(list4), [7e3, %]] :
subs(r = a + 0.1, t = 10^4, T·10) :   evalf(%); list4 ≔ [op(list4), [1e4, %]] :
subs(r = a + 0.1, t = 1.4e4, T·10) :   evalf(%); list4 ≔ [op(list4), [1.4e4, %]] :
subs(r = a + 0.1, t = 2·10^4, T·10) : evalf(%); list4 ≔ [op(list4), [2e4, %]] :
subs(r = a + 0.1, t = 3·10^4, T·10) : evalf(%); list4 ≔ [op(list4), [3e4, %]] :
subs(r = a + 0.1, t = 5·10^4, T·10) : evalf(%); list4 ≔ [op(list4), [5e4, %]] :
subs(r = a + 0.1, t = 7·10^4, T·10) : evalf(%); list4 ≔ [op(list4), [7e4, %]] :
subs(r = a + 0.1, t = 10^5, T·10) :   evalf(%); list4 ≔ [op(list4), [1e5, %]] :
subs(r = a + 0.1, t = 1.4e5, T·10) :   evalf(%); list4 ≔ [op(list4), [1.4e5, %]] :
subs(r = a + 0.1, t = 2·10^5, T·10) : evalf(%); list4 ≔ [op(list4), [2e5, %]] :
subs(r = a + 0.1, t = 3·10^5, T·10) : evalf(%); list4 ≔ [op(list4), [3e5, %]] :
subs(r = a + 0.1, t = 5·10^5, T·10) : evalf(%); list4 ≔ [op(list4), [5e5, %]] :
subs(r = a + 0.1, t = 7·10^5, T·10) : evalf(%); list4 ≔ [op(list4), [7e5, %]] :
subs(r = a + 0.1, t = 10^6, T·10) :   evalf(%); list4 ≔ [op(list4), [1e6, %]] :
subs(r = a + 0.1, t = 1.4e6, T·10) :   evalf(%); list4 ≔ [op(list4), [1.4e6, %]] :
subs(r = a + 0.1, t = 2·10^6, T·10) : evalf(%); list4 ≔ [op(list4), [2e6, %]] :
subs(r = a + 0.1, t = 3·10^6, T·10) : evalf(%); list4 ≔ [op(list4), [3e6, %]] :
subs(r = a + 0.1, t = 5·10^6, T·10) : evalf(%); list4 ≔ [op(list4), [5e6, %]] :
subs(r = a + 0.1, t = 7·10^6, T·10) : evalf(%); list4 ≔ [op(list4), [7e6, %]] :
subs(r = a + 0.1, t = 10^7, T·10) :   evalf(%); list4 ≔ [op(list4), [1e7, %]] :
subs(r = a + 0.1, t = 1.4e7, T·10) :   evalf(%); list4 ≔ [op(list4), [1.4e7, %]] :
subs(r = a + 0.1, t = 2·10^7, T·10) : evalf(%); list4 ≔ [op(list4), [2e7, %]] :
subs(r = a + 0.1, t = 3·10^7, T·10) : evalf(%); list4 ≔ [op(list4), [3e7, %]] :
subs(r = a + 0.1, t = 5·10^7, T·10) : evalf(%); list4 ≔ [op(list4), [5e7, %]] :
subs(r = a + 0.1, t = 7·10^7, T·10) : evalf(%); list4 ≔ [op(list4), [7e7, %]] :
subs(r = a + 0.1, t = 10^8, T·10) :   evalf(%); list4 ≔ [op(list4), [1e8, %]] :
```

$subs(r=a+0.1, t=1.4\cdot 10^8, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [1.4e8, \%]]$:
$subs(r=a+0.1, t=2\cdot 10^8, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [2e8, \%]]$:
$subs(r=a+0.1, t=3\cdot 10^8, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [3e8, \%]]$:
$subs(r=a+0.1, t=5\cdot 10^8, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [5e8, \%]]$:
$subs(r=a+0.1, t=7\cdot 10^8, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [7e8, \%]]$:
$subs(r=a+0.1, t=10^9, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [1e9, \%]]$:
$subs(r=a+0.1, t=1.4\cdot 10^9, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [1.4e9, \%]]$:
$subs(r=a+0.1, t=2\cdot 10^9, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [2e9, \%]]$:
$subs(r=a+0.1, t=3\cdot 10^9, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [3e9, \%]]$:
$subs(r=a+0.1, t=5\cdot 10^9, T\cdot 10)$: $evalf(\%)$; $list4 := [op(list4), [5e9, \%]]$:

112.40822442004750892
112.40608450140762828
112.40287476540220263
112.39859538235975058
112.39217687552589043
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112.36009455762186208
112.33871580146975867
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7.6851261302520472624
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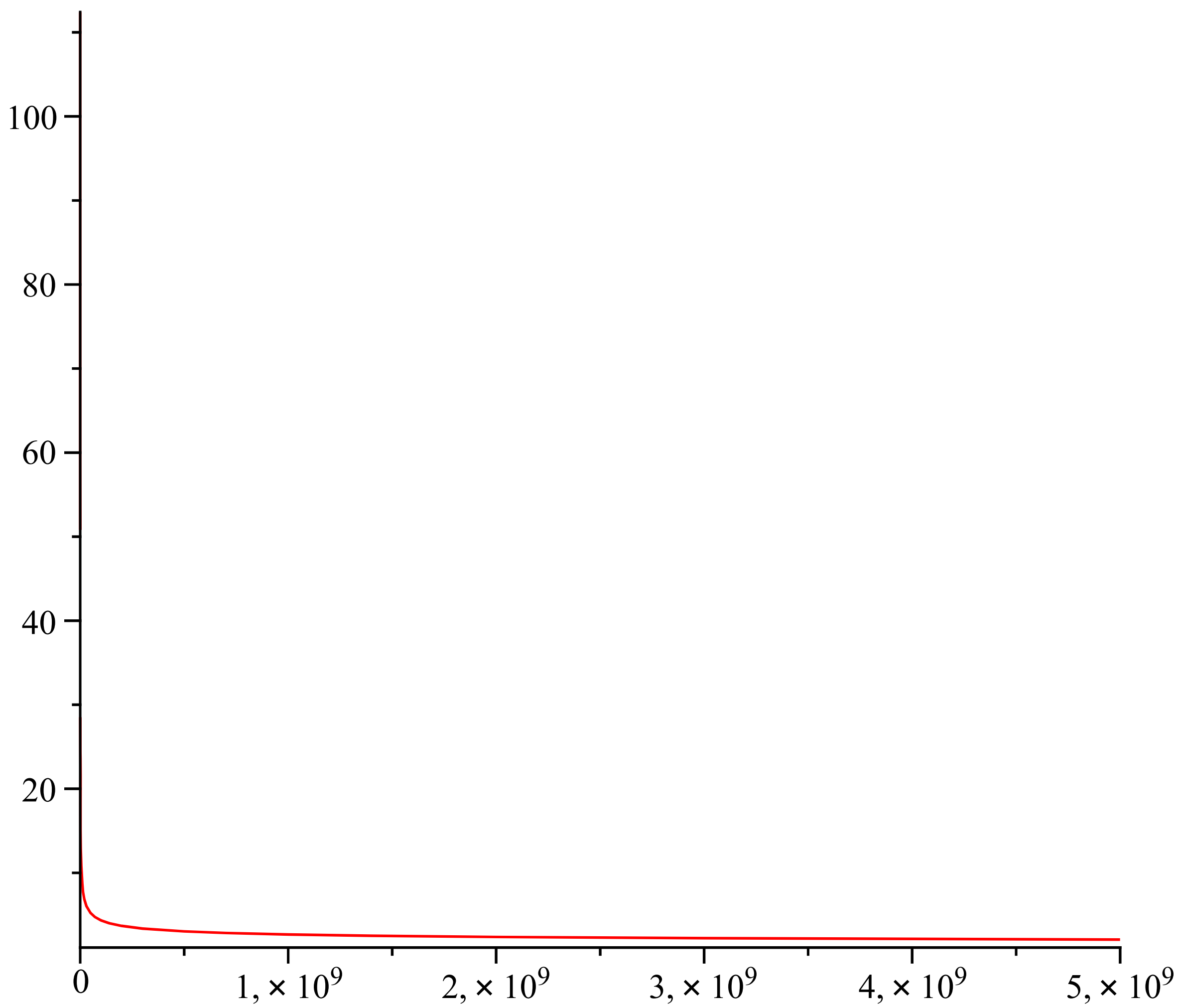
6.0458862757484155270
5.2183701316495190798
4.7636984467956557302
4.3458892053379347325
4.0029067729028639986
3.6851252734940004854
3.3719917098995876643
3.0373786293767533983
2.8471301099689422920
2.6673278661443249548
2.5155032175990419231
2.3708448606122813021
2.2239278638609170556
2.0612991576833486601 **(9)**

> *list4*;

[[5, 112.40822442004750892], [7, 112.40608450140762828], [10., 112.40287476540220263], [14., 112.39859538235975058], [20., 112.39217687552589043], [30., 112.38148087786958005], [50., 112.36009455762186208], [70., 112.33871580146975867], [100., 112.30666184294791226], [140., 112.26394967617932891], [200., 112.19993804129136945], [300., 112.09340274527170743], [500., 111.88089608407511122], [700., 111.66913869630428404], [1000., 111.35290061876362911], [1400., 110.93384303969255631], [2000., 110.31076313001773203], [3000., 109.28677652356500524], [5000., 107.29175716953882746], [7000., 105.36492899428121822], [10000., 102.59649016334113928], [14000., 99.118950346471603060], [20000., 94.321747753524495214], [30000., 87.303766405933937639], [50000., 76.170221768511845192], [70000., 67.857921954661357142], [1. 10^5, 58.858590754640709360], [1.4 10^5, 50.801240895339143582], [2 *e5*, 43.189021303148375479], [3. 10^5, 35.884140610245155859], [5. 10^5, **(10)** 28.518325867095028870], [7. 10^5, 24.588855847536215990], [1. 10^6, 21.077754109393872260], [1.4 10^6, 18.285307107099948069], [2. 10^6, 15.787332043672839658], [3. 10^6, 13.428182650414619410], [5. 10^6, 11.044113673056288559], [7. 10^6, 9.7634724362445681785], [1. 10^7, 8.6105722556808696772], [1.4 10^7, 7.6851261302520472624], [2. 10^7, 6.8480168683079155722], [3. 10^7, 6.0458862757484155270], [5. 10^7, 5.2183701316495190798], [7. 10^7, 4.7636984467956557302], [1. 10^8, 4.3458892053379347325], [1.4 10^8, 4.0029067729028639986], [2. 10^8, 3.6851252734940004854], [3. 10^8, 3.3719917098995876643], [5. 10^8, 3.0373786293767533983], [7. 10^8, 2.8471301099689422920], [1. 10^9, 2.6673278661443249548], [1.4 10^9, 2.5155032175990419231], [2. 10^9, 2.3708448606122813021], [3. 10^9, 2.2239278638609170556], [5. 10^9, 2.0612991576833486601]]

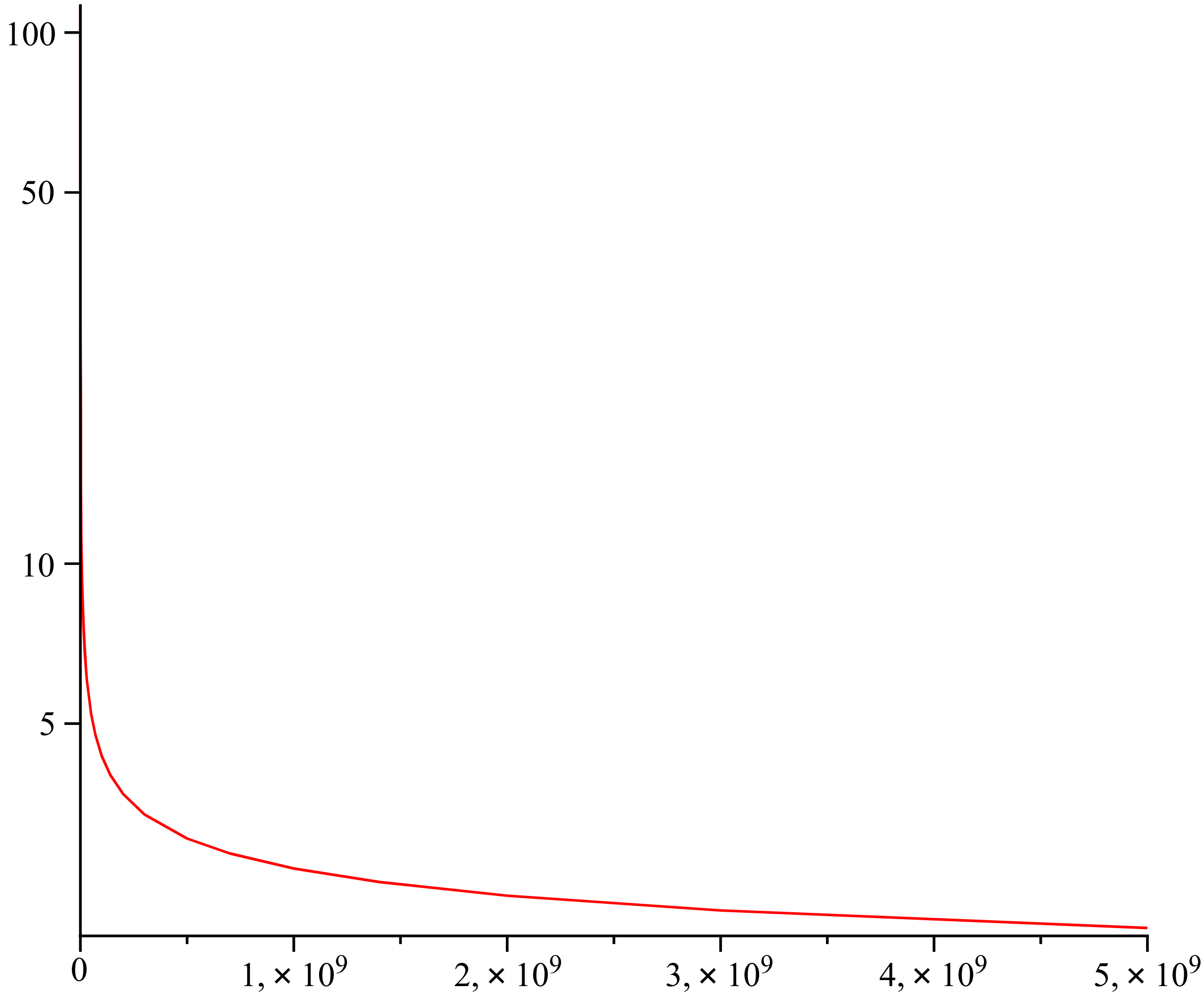
\>

\> *plot*(*list4*)



> *with*(*plots*) :

> *logplot*(*list4*);



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