Temperaturfunktion über und unter einer beheizten Ebene mit radioaktivem Müll

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Von einer horizontalen Bohrung mit D = 14 m Durchmesser gehen horizontale Bohrungen nach beiden Seiten. Ihre Länge beträgt bis zu 1110 m, wobei die ersten und letzten 20 m nicht mit Behältern befüllt werden. Die Behälter liegen in einem Abstand von 0,96 m und geben zu Beginn jeweils 56,25 W Wärme ab. Da der radioaktive Zerfall mit einer Exponentialfunktion abnimmt, ist die Wärmeleistung Q=Q0 * exp(-b*t). Alle Bohrungen verlaufen parallel zueinander und sind in einem Abstand von 29,86 m. Insgesamt sind 0.9 Millionen Behälter mit jeweils 20 kg verglastem radiaoaktivem Müll aus der Wiederaufbereitung zu deponieren. Nach aktueller Planung stehen dafür 27,6 km² zur Verfügung. Die Berechnung erfolgt so, dass von einer ebenen Fläche die freigesetzte Wärme senkrecht zur Ebene in beide Halbräume abfließt. Die Vorgehensweise ist angelehnt an das Lehrbuch Wärme- und Stoffübertrtagung von Hans-Dieter Baehr und Karl Stephan .

```
> restart :
> Qges := 0.9e6 · 56.25;   # gesammter Wärmestrom zu Beginn
```

$$Qges := 5.0625\ 10^{7} \tag{1}$$

```
> q0 := 56.25/(2·(0.96·29.86)); # Wärmestrom pro Quadratmeter in einen Halbraum
  #    Der Abstand in einem Bohrloch ist 0.96 m zwischen den Bohrungen 29.86 m
```

$$q0 := 0.9811411587 \tag{2}$$

```
> EQ1 := diff(T(x, t), t) = a·diff(diff(T(x, t), x), x);
```

$$EQ1 := \frac{\partial}{\partial t} T(x,t) = a \left(\frac{\partial^2}{\partial x^2} T(x,t) \right) \tag{3}$$

```
> with(inttrans) :
> EQ1t := laplace(EQ1, t, s);
```

$$EQ1t := s\ laplace(T(x,t), t, s) - T(x,0) = a \left(\frac{\partial^2}{\partial x^2} laplace(T(x,t), t, s) \right) \tag{4}$$

```
> EQ1t2 := subs(laplace(T(x, t), t, s) = U(x, s), EQ1t);
```

$$EQ1t2 := s\,U(x,s) - T(x,0) = a \left(\frac{\partial^2}{\partial x^2} U(x,s) \right) \tag{5}$$

```
> Tsol1 := dsolve(EQ1t2, U(x, s));
```

$$Tsol1 := U(x,s) = e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}\ _F2(s) + e^{-\frac{\sqrt{s}\,x}{\sqrt{a}}}\ _F1(s) - \frac{1}{2}\ \frac{\left(\left(\int e^{-\frac{\sqrt{s}\,x}{\sqrt{a}}} T(x,0)\,dx \right) e^{\frac{2\sqrt{s}\,x}{\sqrt{a}}} - \left(\int e^{\frac{\sqrt{s}\,x}{\sqrt{a}}} T(x,0)\,dx \right) \right) e^{-\frac{\sqrt{s}\,x}{\sqrt{a}}}}{\sqrt{a}\,\sqrt{s}} \tag{6}$$

```
> Tsol2 := expand(subs(T(x, 0) = 0, Tsol1));
```

(7)

$$Tsol2 := U(x,s) = e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}\,_F2(s) + \frac{_F1(s)}{e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}} \tag{7}$$

Randbedingung bei x= ∞ --> Tsol2=0 --> _F2(s)=0

> $Tsol3 := subs(_F2(s) = 0, Tsol2)$

$$Tsol3 := U(x,s) = \frac{_F1(s)}{e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}} \tag{8}$$

Randbedingung q=q0*exp(-b*t)

> $q := q0 * \exp(-b * t);$

$$q := 0.9811411587\, e^{-bt} \tag{9}$$

> $EQ2 := -\text{lambda}\cdot diff(T(x,t), x) = q;$

$$EQ2 := -\lambda \left(\frac{\partial}{\partial x} T(x,t) \right) = 0.9811411587\, e^{-bt} \tag{10}$$

> $EQ2t := laplace(EQ2, t, s);$

$$EQ2t := -1.\,\lambda \left(\frac{\partial}{\partial x} laplace(T(x,t), t, s) \right) = \frac{0.9811411587}{(s+b)^{1.}} \tag{11}$$

> $EQ2t2 := subs(laplace(T(x,t), t, s) = U(x,s), EQ2t);$

$$EQ2t2 := -1.\,\lambda \left(\frac{\partial}{\partial x} U(x,s) \right) = \frac{0.9811411587}{(s+b)^{1.}} \tag{12}$$

> $EQ2t3 := subs(Tsol3, EQ2t2);$

$$EQ2t3 := -1.\,\lambda \left(\frac{\partial}{\partial x} \left(\frac{_F1(s)}{e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}} \right) \right) = \frac{0.9811411587}{(s+b)^{1.}} \tag{13}$$

> $expand(\%);$

$$\frac{1.\,\lambda\,_F1(s)\,\sqrt{s}}{e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}\,\sqrt{a}} = \frac{0.9811411587}{(s+b)^{1.}} \tag{14}$$

> $F1 := solve\left(subs(x = 0, \%), _F1(s)\right);$

$$F1 := \frac{0.9811411587\,\sqrt{a}}{\sqrt{s}\,\lambda\,(s+b)} \tag{15}$$

> $Tsol4 := subs(_F1(s) = F1, rhs(Tsol3));$

$$Tsol4 := \frac{0.9811411587\,\sqrt{a}}{e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}\,\sqrt{s}\,\lambda\,(s+b)} \tag{16}$$

> $Tsol5 := factor(Tsol4);$

(17)

$$Tsol5 := \frac{0.9811411587\sqrt{a}}{e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}\sqrt{s}\,\lambda\,(s+b)} \tag{17}$$

> $G := q0 * \mathrm{sqrt}(a) / (\exp(\mathrm{sqrt}(s) * x / \mathrm{sqrt}(a)) * \mathrm{sqrt}(s) * \mathrm{lambda});\ H := 1/(s+b);$

$$G := \frac{0.9811411587\sqrt{a}}{e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}\sqrt{s}\,\lambda}$$

$$H := \frac{1}{s+b} \tag{18}$$

> $g := invlaplace(G, s, t);$

$$g := \frac{0.9811411587\sqrt{a}\ invlaplace\left(\frac{1}{e^{\frac{\sqrt{s}\,x}{\sqrt{a}}}\sqrt{s}}, s, t\right)}{\lambda} \tag{19}$$

> $h := invlaplace(H, s, t);$

$$h := e^{-bt} \tag{20}$$

>

Die Funktion wird in die Faktoren G(s,x) und H(s,x) zerlegt. Deren inverse Laplacetransformierte sind g(t,x) und h(t,x).

1. Für $G(s) = \frac{e^{-k\sqrt{s}}}{\sqrt{s}}$:

$$\mathcal{L}^{-1}\left\{\frac{e^{-k\sqrt{s}}}{\sqrt{s}}\right\} = \frac{1}{\sqrt{\pi t}}e^{-\frac{k^2}{4t}}$$

Nennen wir dies $g(t)$.

2. Für $H(s) = \frac{1}{s+b}$:

$$\mathcal{L}^{-1}\left\{\frac{1}{s+b}\right\} = e^{-bt}$$

Nennen wir dies $h(t)$.

>

Der Faltungssatz besagt: $\mathcal{L}^{-1}\{G(s)H(s)\} = (g * h)(t) = \int_0^t g(\tau)h(t-\tau)\,d\tau.$

Setzen wir unsere Funktionen ein:

$$f(t) = \int_0^t \frac{1}{\sqrt{\pi\tau}} e^{-\frac{k^2}{4\tau}} \cdot e^{-b(t-\tau)}\,d\tau$$

Ziehen wir die Konstante e^{-bt} aus dem Integral:

$$f(t) = \frac{e^{-bt}}{\sqrt{\pi}} \int_0^t \tau^{-1/2} e^{-\frac{k^2}{4\tau}+b\tau}\,d\tau$$

```
> ft := exp(-b·t)/sqrt(π)·int(τ^(-1/2)·exp(-k^2/(4·τ)+b·τ), τ=0..t);
```

$$ft := \frac{e^{-bt}\left(\int_0^t \frac{e^{-\frac{1}{4}\frac{k^2}{\tau}+b\tau}}{\sqrt{\tau}}\,d\tau\right)}{\sqrt{\pi}} \quad (21)$$

```
> integrand := τ^(-1/2)·exp(-k^2/(4·τ)+b·τ);
```

$$integrand := \frac{e^{-\frac{1}{4}\frac{k^2}{\tau}+b\tau}}{\sqrt{\tau}} \quad (22)$$

```
> gl1 := sqrt(tau) = y;
```

$$gl1 := \sqrt{\tau} = y \quad (23)$$

```
> gl2 := solve(gl1, tau);
```

$$gl2 := y^2 \quad (24)$$

```
> integrand2 := 2·e^(-(k^2/(4·y^2)-b·y^2));
```

$$integrand2 := 2\,e^{-\frac{1}{4}\frac{k^2}{y^2}+by^2} \quad (25)$$

```
> int(2·e^(-(k^2/(4·y^2)-b·y^2)), y=0..sqrt(t));
```

$$\int_0^{\sqrt{t}} 2\,e^{-\frac{1}{4}\frac{k^2}{y^2}+by^2}\,dy \quad (26)$$

```
> y := 'y': xi := 'xi': b := 'b': a := 'a': lambda := 'lambda': q0 := 'q0':
> restart:
> f := -I*q0*sqrt(a)/lambda*exp(-b*t)/(2*sqrt(b))*(+exp(-I*y)*erfc(xi-I*sqrt(b
```

```
*t))-exp(I*y)*erfc(xi+I*sqrt(b*t)));
```

$$f := -\frac{\frac{1}{2}\,\mathrm{I}\,q0\,\sqrt{a}\;\mathrm{e}^{-b\,t}\left(\mathrm{e}^{-\mathrm{I}y}\,\mathrm{erfc}\left(\xi-\mathrm{I}\sqrt{b\,t}\right)-\mathrm{e}^{\mathrm{I}y}\,\mathrm{erfc}\left(\xi+\mathrm{I}\sqrt{b\,t}\right)\right)}{\lambda\sqrt{b}} \tag{27}$$

```
> y := x·sqrt(b)/sqrt(a); xi := x/sqrt(4·a·t); f;
```

$$y := \frac{x\sqrt{b}}{\sqrt{a}}$$

$$\xi := \frac{1}{2}\,\frac{x}{\sqrt{a\,t}}$$

$$-\frac{1}{\lambda\sqrt{b}}\left(\frac{1}{2}\,\mathrm{I}\,q0\,\sqrt{a}\;\mathrm{e}^{-b\,t}\left(\mathrm{e}^{-\frac{\mathrm{I}x\sqrt{b}}{\sqrt{a}}}\,\mathrm{erfc}\left(\frac{1}{2}\,\frac{x}{\sqrt{a\,t}}-\mathrm{I}\sqrt{b\,t}\right)-\mathrm{e}^{\frac{\mathrm{I}x\sqrt{b}}{\sqrt{a}}}\,\mathrm{erfc}\left(\frac{1}{2}\,\frac{x}{\sqrt{a\,t}}+\mathrm{I}\sqrt{b\,t}\right)\right)\right) \tag{28}$$

```
> with(inttrans);
```

[*addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable*] (29)

```
> F := sqrt(π/(4·B))·(exp(-2·sqrt(A·B))·erfc(sqrt(B·t)-sqrt(A/t))+exp(2·sqrt(A·B))
  ·erfc(sqrt(B·t)+sqrt(A/t)));
```

$$F := \frac{1}{2}\sqrt{\frac{\pi}{B}}\left(\mathrm{e}^{-2\sqrt{AB}}\,\mathrm{erfc}\left(\sqrt{B\,t}-\sqrt{\frac{A}{t}}\right)+\mathrm{e}^{2\sqrt{AB}}\,\mathrm{erfc}\left(\sqrt{B\,t}+\sqrt{\frac{A}{t}}\right)\right) \tag{30}$$

```
> A := x^2/(4·a); B := -b;
```

$$A := \frac{1}{4}\,\frac{x^2}{a}$$

$$B := -b \tag{31}$$

```
> F1 := q0·exp(-b·t)·simplify(F);
```

$$F1 := -\frac{1}{2}\,q0\,\mathrm{e}^{-b\,t}\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\left(-\mathrm{e}^{-\sqrt{-\frac{x^2 b}{a}}}-\mathrm{e}^{-\sqrt{-\frac{x^2 b}{a}}}\,\mathrm{erf}\left(-\sqrt{-b\,t}+\frac{1}{2}\sqrt{\frac{x^2}{a\,t}}\right)-\mathrm{e}^{\sqrt{-\frac{x^2 b}{a}}}+\mathrm{e}^{\sqrt{-\frac{x^2 b}{a}}}\,\mathrm{erf}\left(\sqrt{-b\,t}+\frac{1}{2}\sqrt{\frac{x^2}{a\,t}}\right)\right) \tag{32}$$

```
> b := 7.3125e-10; #1/Sekunde  Zerfallsrate
```

$$b := 7.3125\ 10^{-10} \tag{33}$$

Die Stoffdaten von Tonstein dürften ähnlich sein wie die von Ton.
Aus dem Lehrbuch von Baehr/Stephan stammen die folgenden Daten.

> rho := 1450; *cp* := 880; # *Dichte in* $\frac{kg}{m3}$, *spezifische Wärmekapazität in* $\frac{J}{(kg \cdot K)}$

$$\rho := 1450$$

$$cp := 880 \quad (34)$$

> lambda := 1.28; $a := \frac{\text{lambda}}{\text{rho} \cdot cp}$; # *lambda in* $\frac{W}{(m \cdot K)}$, *a in* $\frac{m^2}{s}$

$$\lambda := 1.28$$

$$a := 0.000001003134796 \quad (35)$$

> $y := x \cdot \text{sqrt}\left(\frac{b}{a}\right)$; $\text{xi} := \frac{x}{\text{sqrt}(4 \cdot a \cdot t)}$;

$$y := 0.02699934895\, x$$

$$\xi := \frac{499.2181388\, x}{\sqrt{t}} \quad (36)$$

> *f*;

$$-14.46794146\, I\, q0\, e^{-7.3125\, 10^{-10} t} \left(e^{-0.02699934895\, I x} \operatorname{erfc}\left(\frac{499.2181388\, x}{\sqrt{t}} - 0.00002704163457\, I \sqrt{t} \right) - e^{0.02699934895\, I x} \operatorname{erfc}\left(\frac{499.2181388\, x}{\sqrt{t}} + 0.00002704163457\, I \sqrt{t} \right) \right) \quad (37)$$

> *Qges* := 0.9e6 · 56.25; # *gesammter Wärmestrom zu Beginn*

$$Qges := 5.0625\, 10^7 \quad (38)$$

> $q0 := \frac{56.25}{2 \cdot (0.96 \cdot 29.86)}$;

Das ist die halbe Wärmestromdichte eines Behälters bezogen auf die Fläche.
Denn nur die Hälfte geht in die eine Richtung.

$$q0 := 0.9811411587 \quad (39)$$

> *f*

$$-14.19509285\, I\, e^{-7.3125\, 10^{-10} t} \left(e^{-0.02699934895\, I x} \operatorname{erfc}\left(\frac{499.2181388\, x}{\sqrt{t}} - 0.00002704163457\, I \sqrt{t} \right) - e^{0.02699934895\, I x} \operatorname{erfc}\left(\frac{499.2181388\, x}{\sqrt{t}} + 0.00002704163457\, I \sqrt{t} \right) \right) \quad (40)$$

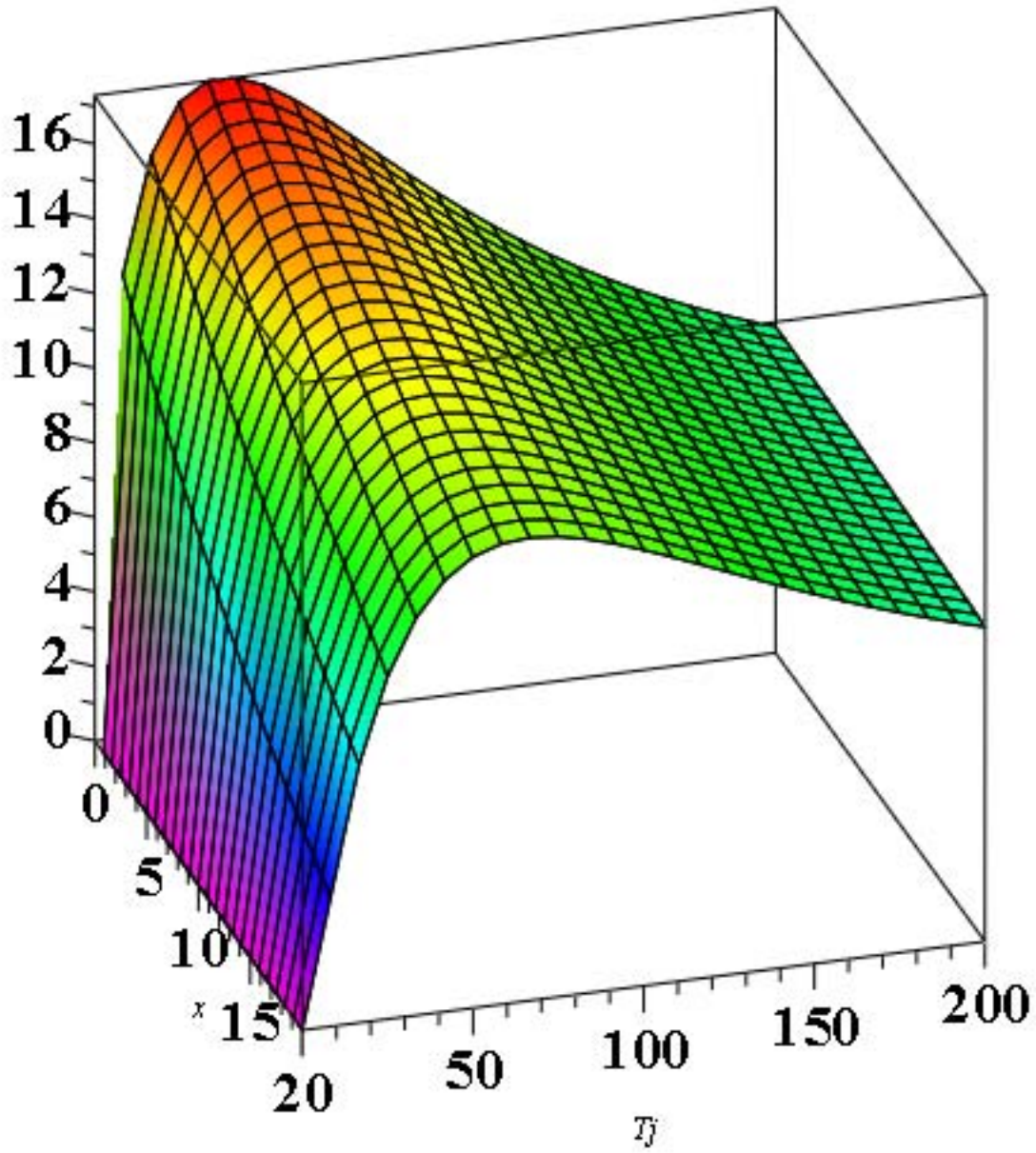
> *with*(*plots*) :

> *t* := *Tj* · (365.25 · 24 · 3600); *Tj* :='*Tj*':

$$t := 3.155760000\, 10^7\, Tj \quad (41)$$

> *plot3d*(*f*, *x* = 0 ..20, Tj = 0 ..200, *orientation* = [- 17, 65], *axes* = *boxed*, *axesfont* = [*times* , *BOLD*, 18], *title* = "Temperaturfeld T in K, x in m, t in a bis 200 Jahre", *font* = [*times* , *BOLD*, 18], *color* = *Z*(*Hue*));

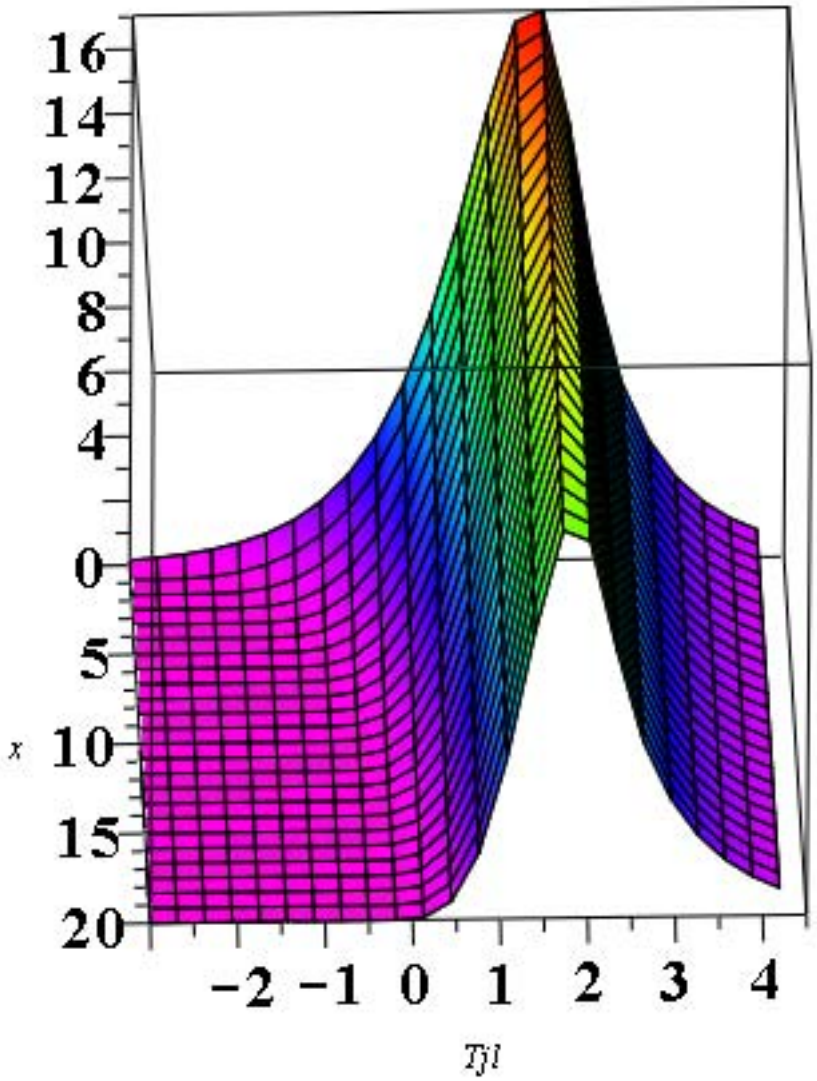
Temperaturfeld T in K, x in m, t in a bis 200 Jahre




```
> Tj := 10^Tjl :
> plot3d(f, x = 0 ..20, Tjl = -3 ..4.5, orientation = [ -17, 65 ], axes = boxed, axesfont = [times ,
      BOLD, 18 ], title = "Temperaturfeld T in K, x in m, t in a, logarithmisch", font = [times ,
```

BOLD, 18], *color* = *Z*(*Hue*));

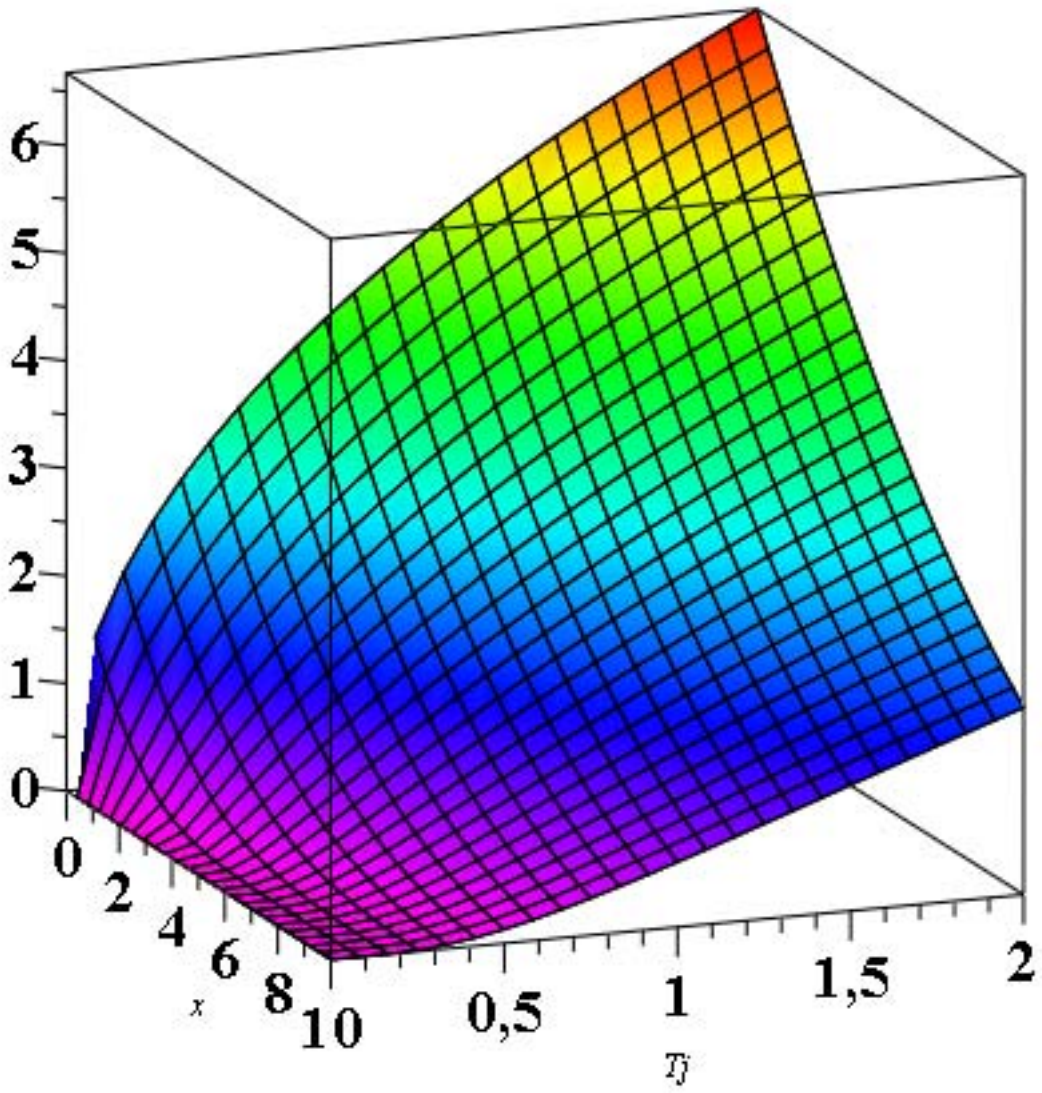
Temperaturfeld T in K, x in m, t in a, logarithmisch



> *Tj* := 'Tj':

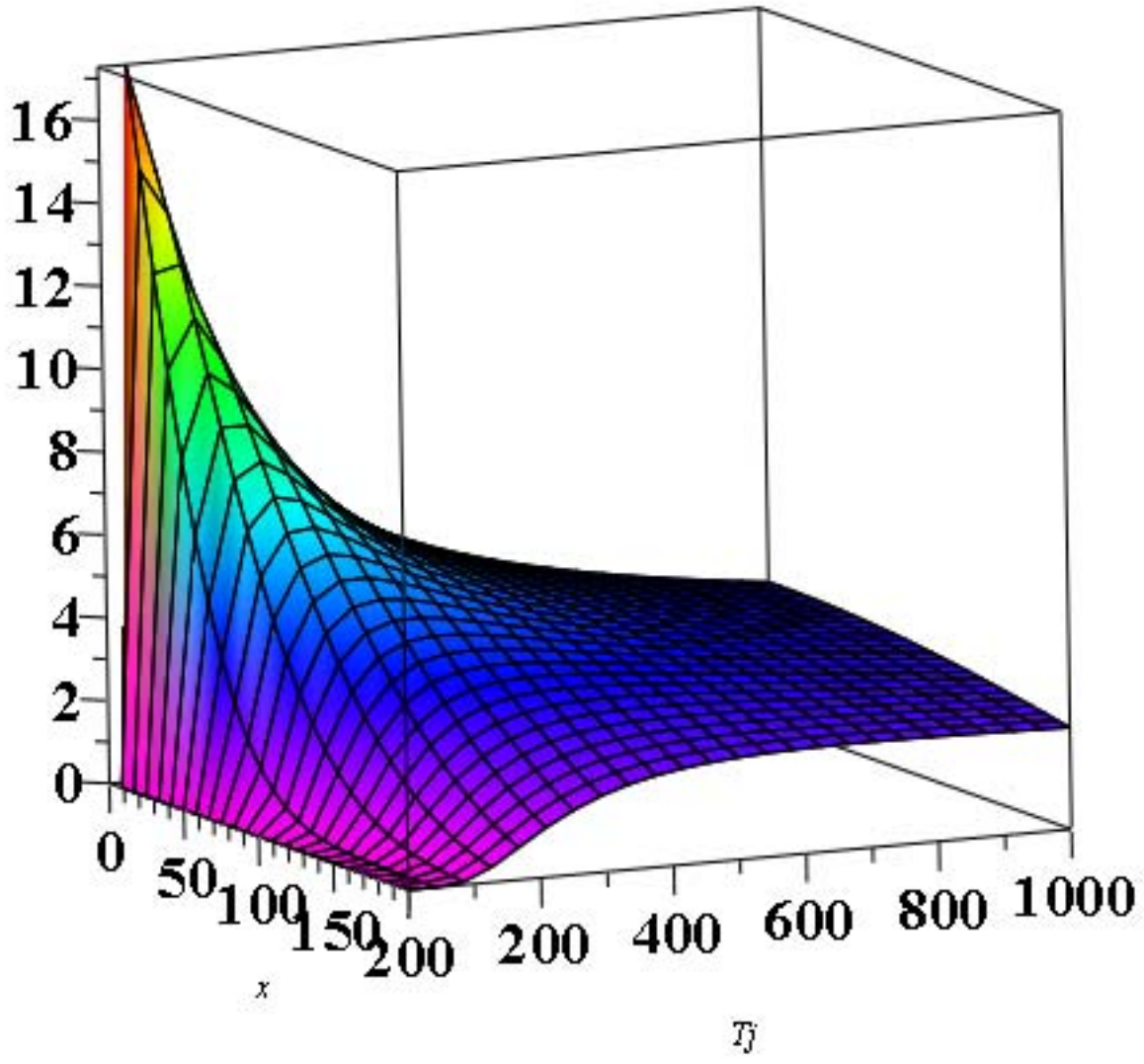
> *plot3d*(*f*, *x* = 0 ..10, *Tj* = 0 ..2, *orientation* = [−21, 76], *axes* = *boxed*, *title* = "Temperaturfeld T in K, x in m, t in a bis 2 Jahre", *font* = [*times* , *BOLD*, 18]);

Temperaturfeld T in K, x in m, t in a bis 2 Jahre



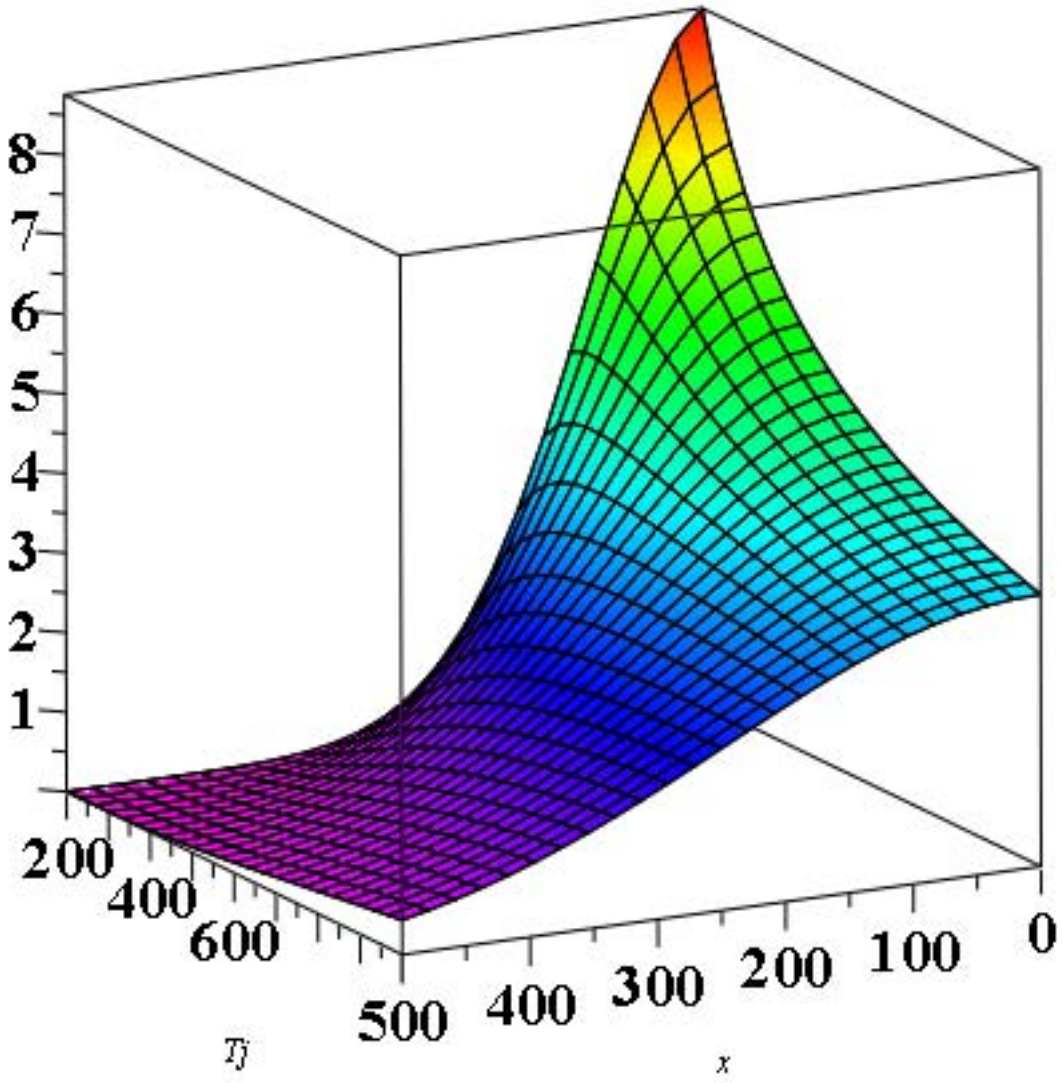

```
> plot3d(f, x = 0 ..200, Tj = 0 ..1000, orientation = [ -21, 76 ], axes = boxed, title
      = "Temperaturfeld T in K, x in m, t in a bis 1000 Jahre", font = [ times , BOLD, 18 ]);
```

Temperaturfeld T in K, x in m, t in a bis 1000 Jahre



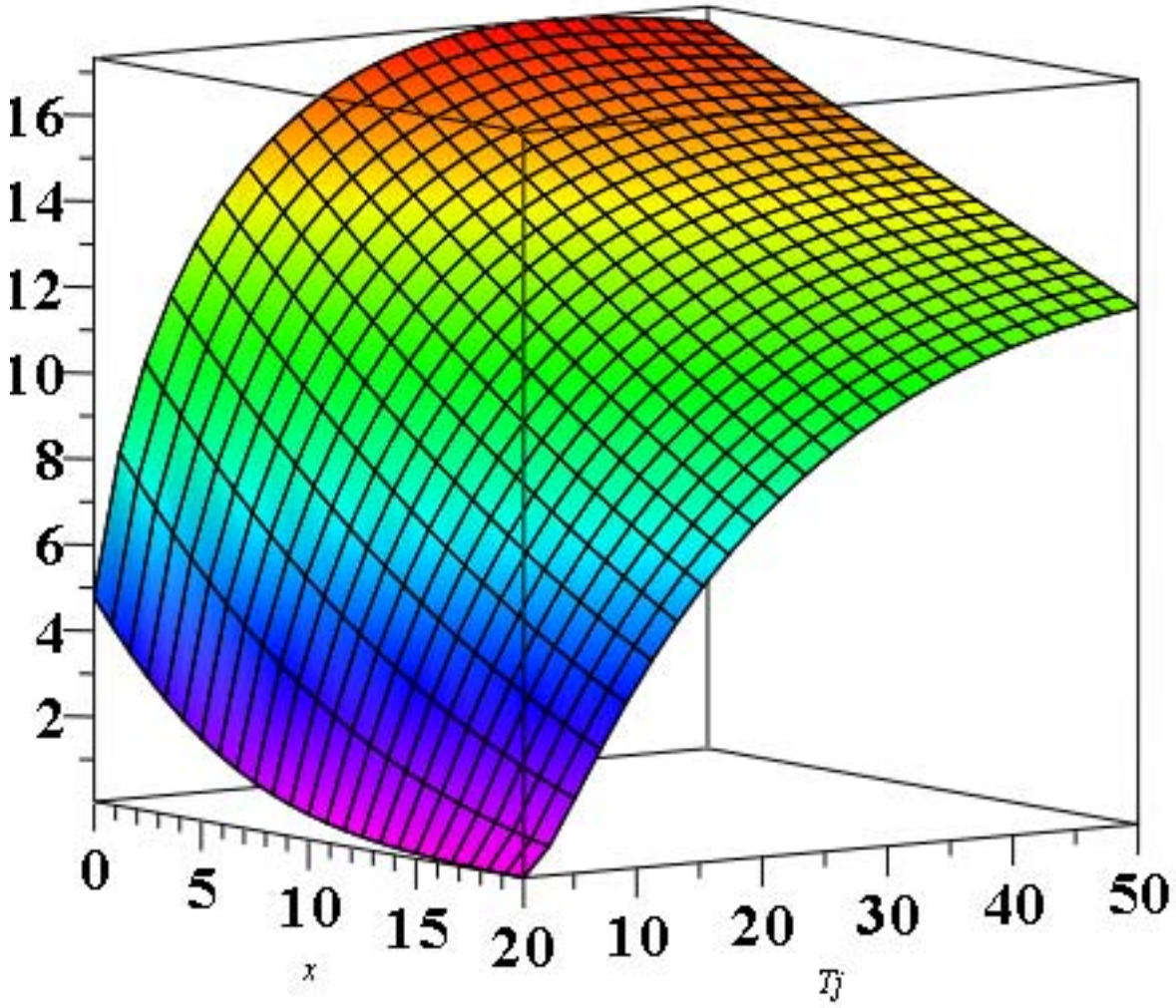

```
> plot3d(f, x = 0 ..500, Tj = 200 ..1000, orientation = [62, 75], axes = boxed, title
      = "Temperaturfeld T in K, x in m, t in a bis 1000 Jahre", font = [times , BOLD, 18]);
```

Temperaturfeld T in K, x in m, t in a bis 1000 Jahre



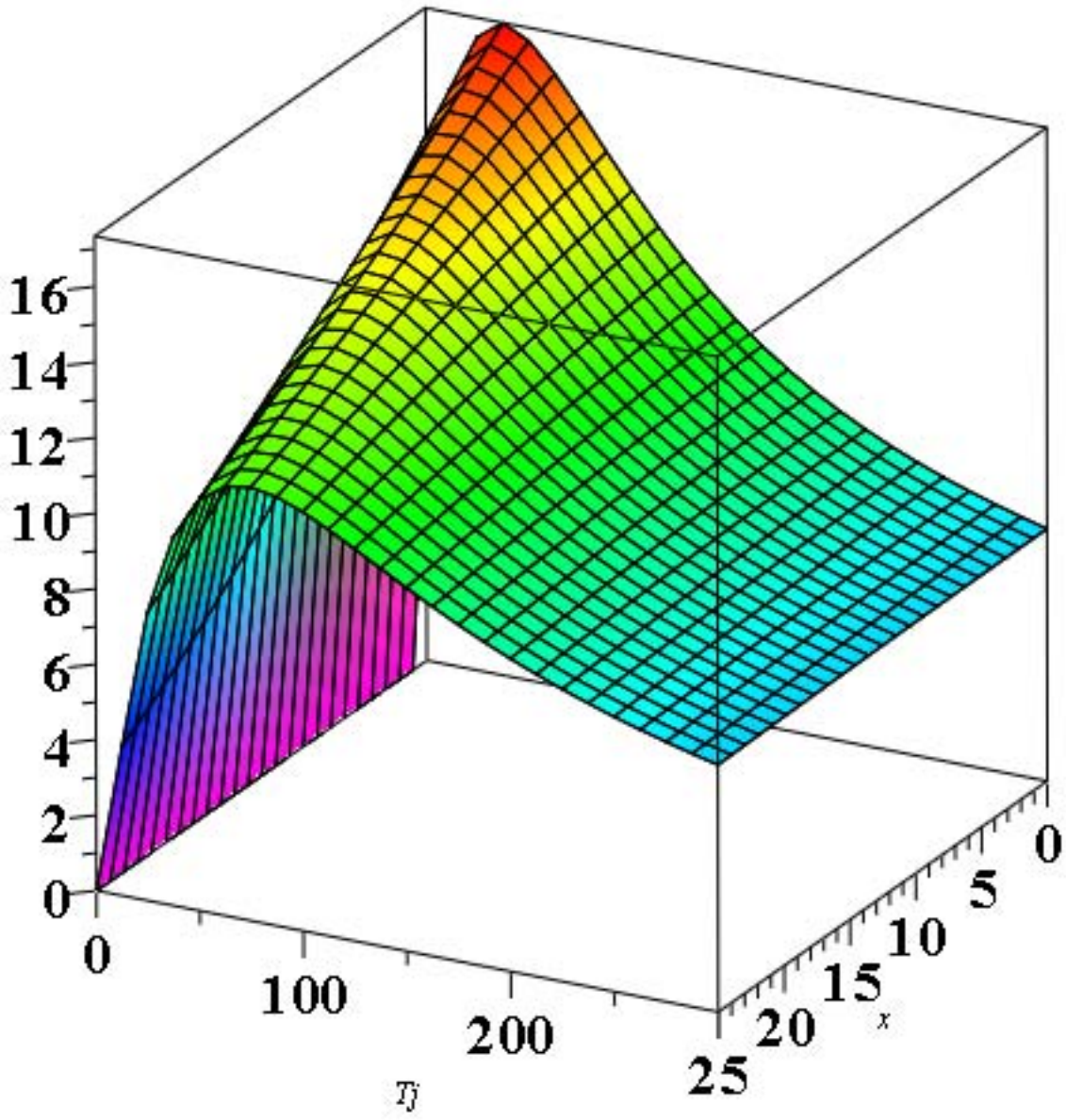

```
> plot3d(f, x = 0 ..20, Tj = 1 ..50, orientation = [ -35, 83 ], axes = boxed, title
      = "Temperaturfeld T in K, x in m, t in a bis 50 Jahre", font = [times , BOLD, 18 ]);
```

Temperaturfeld T in K, x in m, t in a bis 50 Jahre



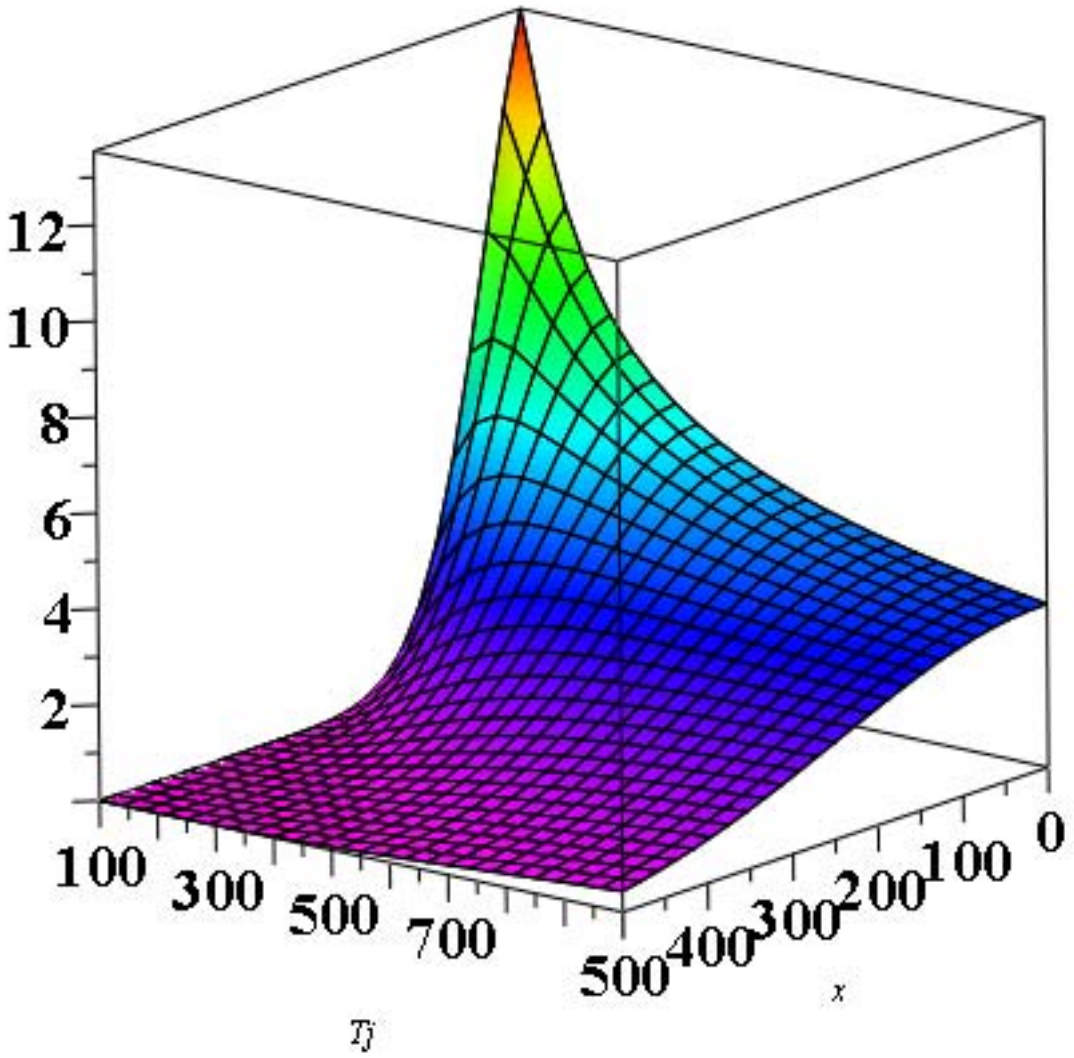

```
> plot3d(f, x = 0 ..25, Tj = 0 ..300, orientation = [28, 68], axes = boxed, title
      = "Temperaturfeld T in K, x in m, Tj in a bis 300 Jahre", font = [times , BOLD, 18]);
```

Temperaturfeld T in K, x in m, Tj in a bis 300 Jahre



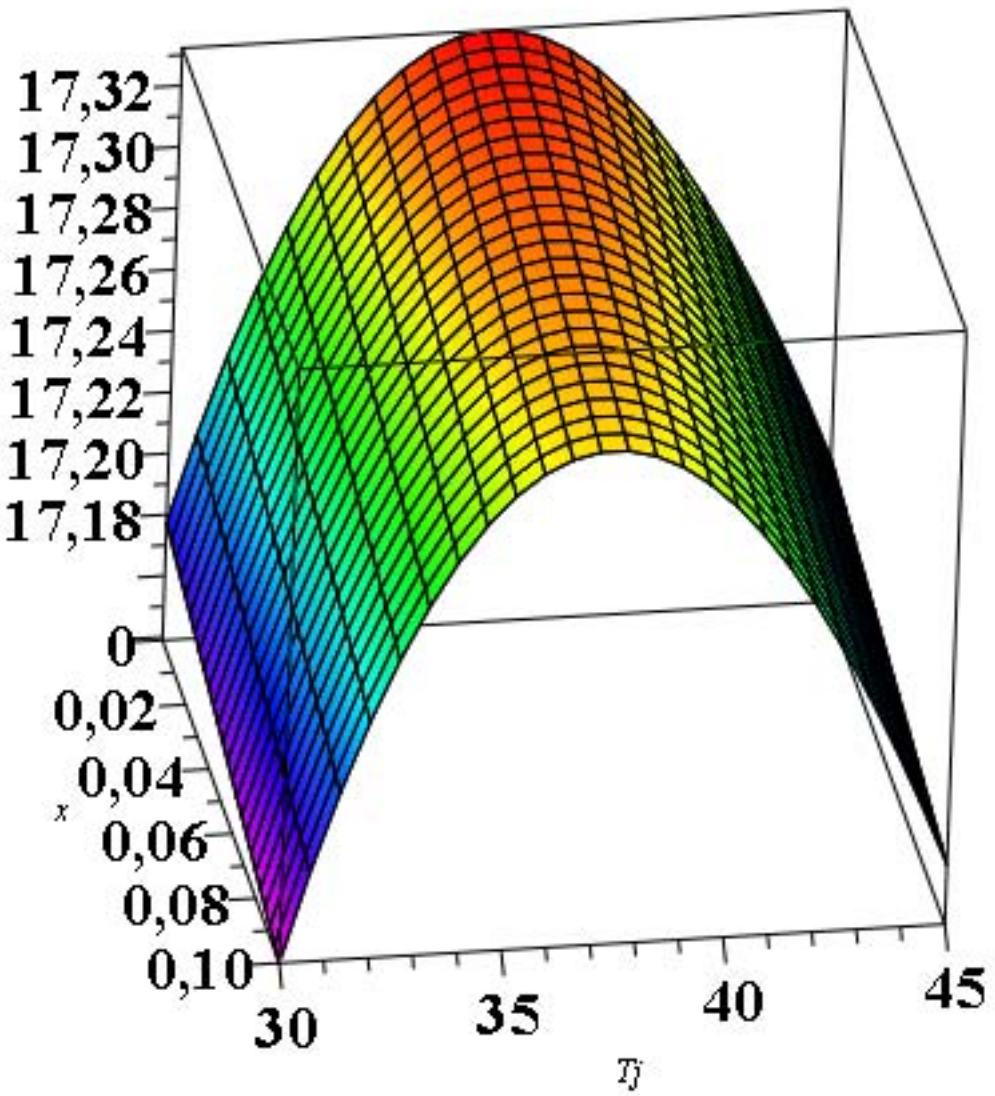

```
> plot3d(f, x = 0 ..500, Tj = 100 ..1000, orientation = [28, 68], axes = boxed, title
      = "Temperaturfeld T in K, x in m, Tj in a,  bis 1000 Jahre", font = [times , BOLD, 18]);
```

Temperaturfeld T in K, x in m, Tj in a, bis 1000 Jahre



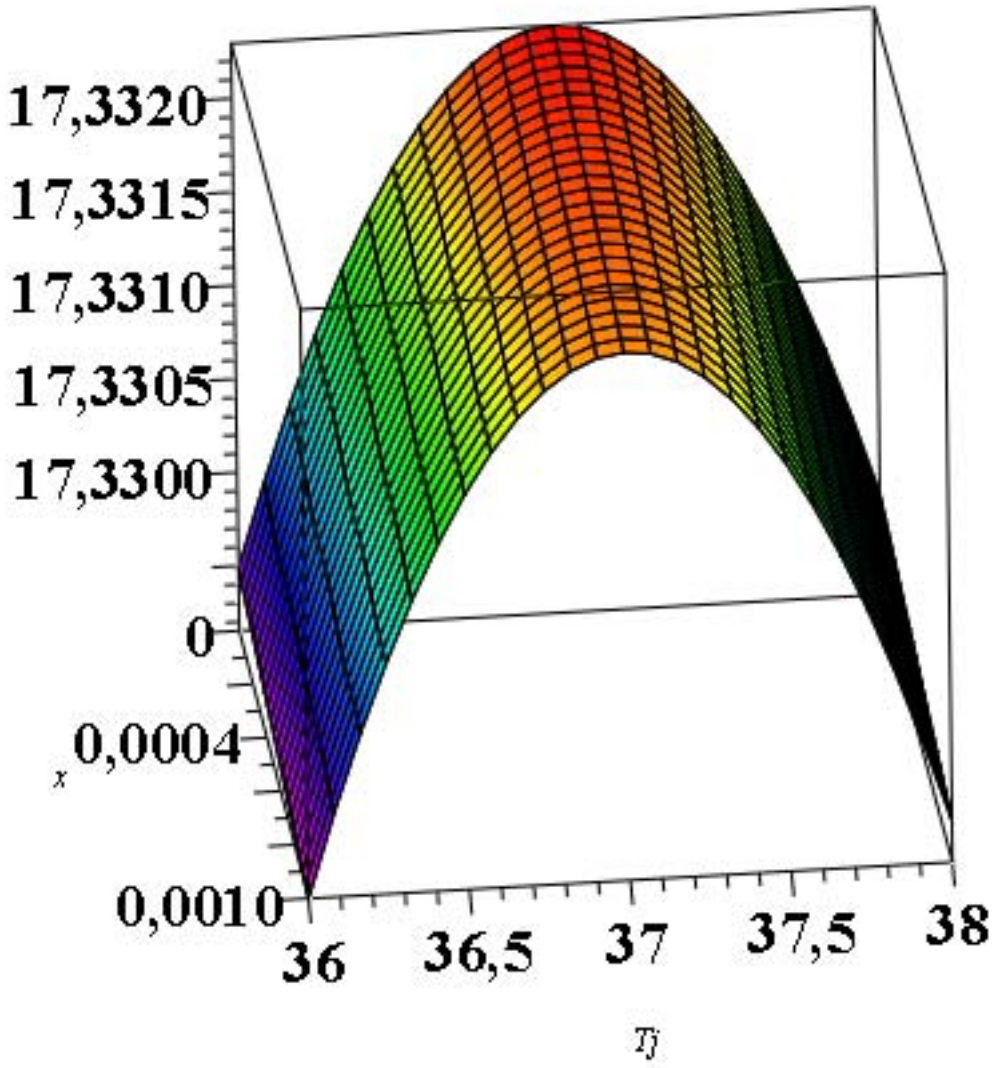

```
> plot3d(f, x = 0 ...1, Tj = 30 ..45, orientation = [ -12, 82 ], axes = boxed, title
      = "Temperaturfeld T in K, x in m, Tj in a,  30 bis 45 Jahre", font = [ times , BOLD, 18 ]);
```

Temperaturfeld T in K, x in m, Tj in a, 30 bis 45 Jahre




```
> plot3d(f, x = 0 ...001, Tj = 36 ..38, orientation = [ -12, 82 ], axes = boxed, title
      = "Temperaturfeld T in K, x in m Maximum bei 17.3323 K nach 37 Jahren", font
      = [ times , BOLD, 18 ]);
```

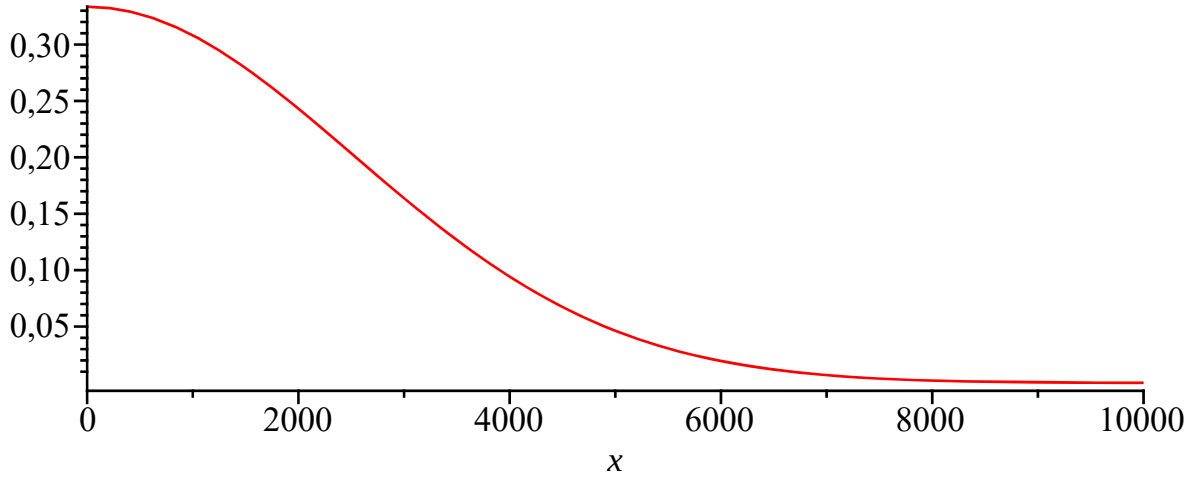
Temperaturfeld T in K, x in m Maximum bei 17.3323 K nach 37 Jahren




```
> f
```

$$-14.19509285\,\mathrm{I}\,e^{-0.02307649500\,Tj}\left(e^{-0.02699934895\,\mathrm{I}x}\,\mathrm{erfc}\left(\frac{0.08886656088\,x}{\sqrt{Tj}}-0.1519094961\,\mathrm{I}\sqrt{Tj}\right)-e^{0.02699934895\,\mathrm{I}x}\,\mathrm{erfc}\left(\frac{0.08886656088\,x}{\sqrt{Tj}}+0.1519094961\,\mathrm{I}\sqrt{Tj}\right)\right) \quad (42)$$

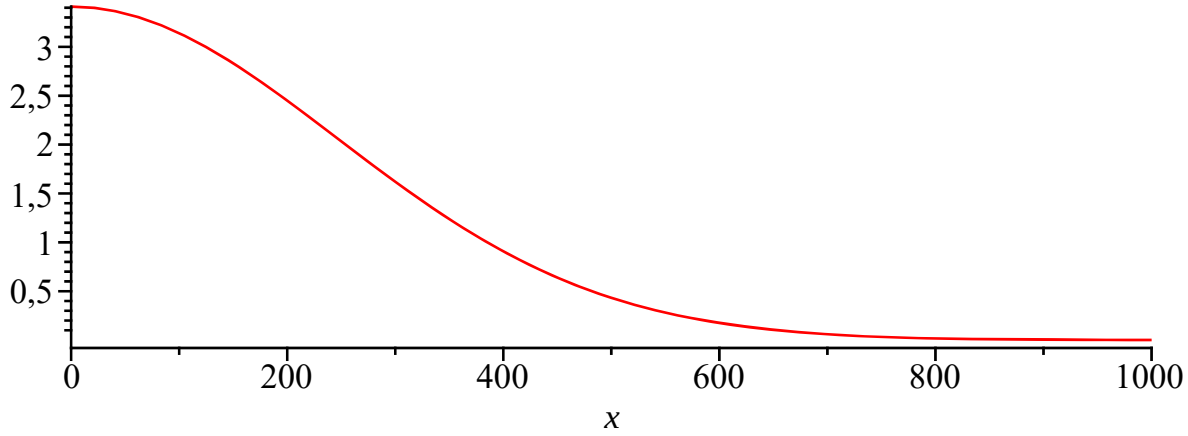
```
> plot(subs(Tj = 100000, f), x = 0 ..10000);
```




```
> evalf(int(subs(Tj = 100000, f), x = 0 ..10000));
```

1051.440302 − 0. I **(43)**

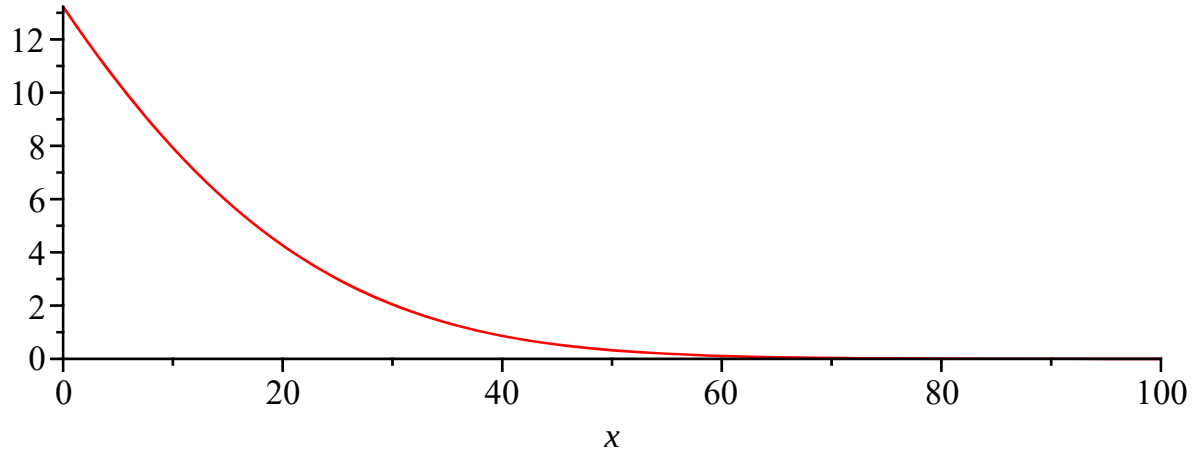
```
> plot(subs(Tj = 1000, f), x = 0 ..1000);
```




```
> evalf(int(subs(Tj = 1000, f), x = 0 ..1000));
```

1051.460139 + 0. I **(44)**

```
> plot(subs(Tj = 10, f), x = 0 ..100);
```




```
> evalf(int(subs(Tj = 10, f), x = 0 ..100));
```

216.6879213 + 0. I **(45)**

```
> Fint := 0 :
  x1 := 0 : x2 := 1 : x3 := 10 : x4 := 100 : x5 := 1000 :
  x6 := 10000 : x7 := 100000 : x8 := 1000000 :
  Fint[1] := evalf(subs(Tj = 100000, int(f, x = x1..x2)));
  Fint[2] := evalf(subs(Tj = 100000, int(f, x = x2..x3)));
  Fint[3] := evalf(subs(Tj = 100000, int(f, x = x3..x4)));
  Fint[4] := evalf(subs(Tj = 100000, int(f, x = x4..x5)));
  Fint[5] := evalf(subs(Tj = 100000, int(f, x = x5..x6)));
  Fint[6] := evalf(subs(Tj = 100000, int(f, x = x6..x7)));
  Fint[7] := evalf(subs(Tj = 100000, int(f, x = x7..x8)));
  Fint100000 := Fint[1] + Fint[2] + Fint[3] + Fint[4] + Fint[5] + Fint[6] + Fint[7];
```

$$Fint_1 := 0.3335053037 - 0.\,I$$

$$Fint_2 := 3.001539038 - 0.\,I$$

$$Fint_3 := 30.00670591 - 0.\,I$$

$$Fint_4 := 291.5848074 - 0.\,I$$

$$Fint_5 := 726.5137443 - 0.\,I$$

$$Fint_6 := 0.07397569162 - 0.\,I$$

$$Fint_7 := 8.135643873\ 10^{-343} - 0.\,I$$

$$Fint100000 := 1051.514278 - 0.\,I \qquad \textbf{(46)}$$

```
> Fint := 0 :
  x1 := 0 : x2 := 1 : x3 := 10 : x4 := 100 : x5 := 1000 :
  x6 := 10000 : x7 := 100000 : x8 := 1000000 :
  Fint[1] := evalf(subs(Tj = 10000, int(f, x = x1..x2)));
  Fint[2] := evalf(subs(Tj = 10000, int(f, x = x2..x3)));
  Fint[3] := evalf(subs(Tj = 10000, int(f, x = x3..x4)));
  Fint[4] := evalf(subs(Tj = 10000, int(f, x = x4..x5)));
  Fint[5] := evalf(subs(Tj = 10000, int(f, x = x5..x6)));
  Fint[6] := evalf(subs(Tj = 10000, int(f, x = x6..x7)));
  Fint[7] := evalf(subs(Tj = 10000, int(f, x = x7..x8)));
  Fint10000 := Fint[1] + Fint[2] + Fint[3] + Fint[4] + Fint[5] + Fint[6] + Fint[7];
```

$$Fint_1 := 1.056706609 + 0.\,I$$

$$Fint_2 := 9.510082890 + 0.\,I$$

$$Fint_3 := 94.82517435 + 0.\,I$$

$$Fint_4 := 727.5645476 + 0.\,I$$

$$Fint_5 := 218.5572592$$

$$Fint_6 := 2.865783162\ 10^{-33} + 0.\,I$$

$$Fint_7 := 0.$$

$$Fint10000 := 1051.513771 + 0.\,I \qquad \textbf{(47)}$$

```
> Fint := 0 :
  x1 := 0 : x2 := 1 : x3 := 10 : x4 := 100 : x5 := 1000 :
  x6 := 10000 : x7 := 100000 : x8 := 1000000 :
  Fint[1] := evalf(subs(Tj = 1000, int(f, x = x1..x2)));
  Fint[2] := evalf(subs(Tj = 1000, int(f, x = x2..x3)));
```

```
Fint[3] := evalf(subs(Tj = 1000, int(f, x = x3 ..x4)));
Fint[4] := evalf(subs(Tj = 1000, int(f, x = x4 ..x5)));
Fint[5] := evalf(subs(Tj = 1000, int(f, x = x5 ..x6)));
Fint[6] := evalf(subs(Tj = 1000, int(f, x = x6 ..x7)));
Fint1000 := Fint[1] + Fint[2] + Fint[3] + Fint[4] + Fint[5] + Fint[6];
```

$$Fint_1 := 3.411866256 + 0.\,I$$

$$Fint_2 := 30.69747133 + 0.\,I$$

$$Fint_3 := 297.8869018 + 0.\,I$$

$$Fint_4 := 719.4638997 + 0.\,I$$

$$Fint_5 := 0.05358085938$$

$$Fint_6 := 0.$$

$$Fint1000 := 1051.513720 + 0.\,I \qquad \textbf{(48)}$$

```
> Fint := 0:
x1 := 0: x2 := 1: x3 := 10: x4 := 100: x5 := 1000:
x6 := 10000: x7 := 100000: x8 := 1000000:
Fint[1] := evalf(subs(Tj = 100, int(f, x = x1 ..x2)));
Fint[2] := evalf(subs(Tj = 100, int(f, x = x2 ..x3)));
Fint[3] := evalf(subs(Tj = 100, int(f, x = x3 ..x4)));
Fint[4] := evalf(subs(Tj = 100, int(f, x = x4 ..x5)));
Fint[5] := evalf(subs(Tj = 100, int(f, x = x5 ..x6)));
Fint100 := Fint[1] + Fint[2] + Fint[3] + Fint[4] + Fint[5];
```

$$Fint_1 := 13.50626668 + 0.\,I$$

$$Fint_2 := 117.7869824 + 0.\,I$$

$$Fint_3 := 695.4123620 + 0.\,I$$

$$Fint_4 := 120.1879156$$

$$Fint_5 := 1.870058512\ 10^{-34} + 0.\,I$$

$$Fint100 := 946.8935267 + 0.\,I \qquad \textbf{(49)}$$

```
> Fint := 0:
x1 := 0: x2 := 1: x3 := 10: x4 := 100: x5 := 1000:
x6 := 10000: x7 := 100000: x8 := 1000000:
Fint[1] := evalf(subs(Tj = 10, int(f, x = x1 ..x2)));
Fint[2] := evalf(subs(Tj = 10, int(f, x = x2 ..x3)));
Fint[3] := evalf(subs(Tj = 10, int(f, x = x3 ..x4)));
Fint[4] := evalf(subs(Tj = 10, int(f, x = x4 ..x5)));
Fint10 := Fint[1] + Fint[2] + Fint[3] + Fint[4];
```

$$Fint_1 := 12.92445669 + 0.\,I$$

$$Fint_2 := 91.49842153 + 0.\,I$$

$$Fint_3 := 112.2650431 + 0.\,I$$

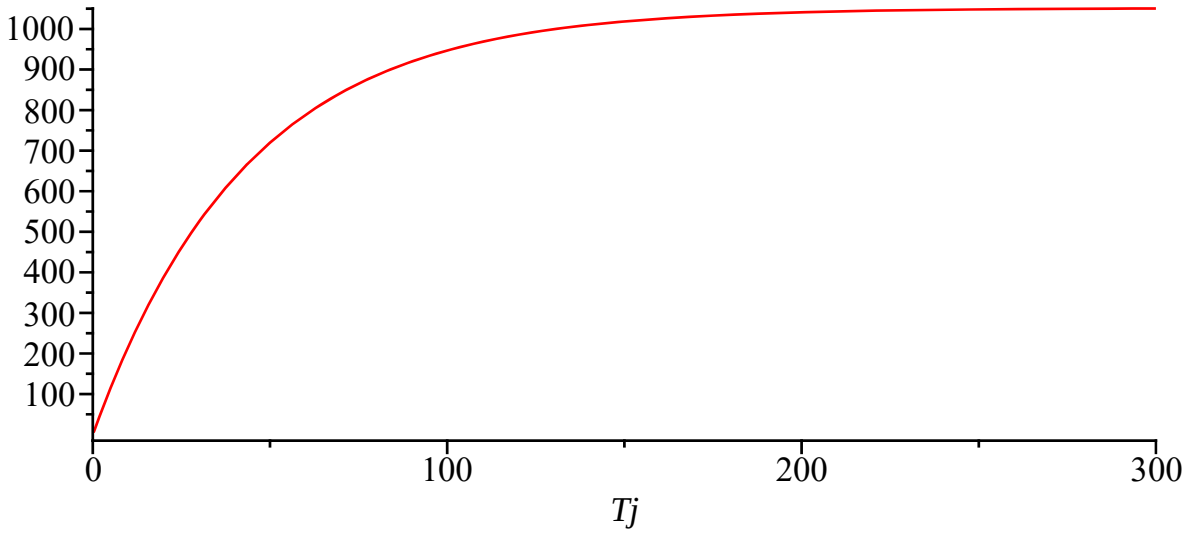
$$Fint_4 := 0.001656613891$$

$$Fint10 := 216.6895779 + 0.\,I \qquad \textbf{(50)}$$

```
>
> FInt := int(f, x = 0 ..10000);
```

$$FInt := \int_0^{10000} \left(-14.19509285\,\mathrm{I}\, e^{-0.02307649500\,Tj} \left(e^{-0.02699934895\,\mathrm{I}\,x} \operatorname{erfc}\left(\frac{0.08886656088\,x}{\sqrt{Tj}} - 0.1519094961\,\mathrm{I}\sqrt{Tj} \right) - e^{0.02699934895\,\mathrm{I}\,x} \operatorname{erfc}\left(\frac{0.08886656088\,x}{\sqrt{Tj}} + 0.1519094961\,\mathrm{I}\sqrt{Tj} \right) \right) \right) dx \quad (51)$$

> plot(FInt, Tj = 0..300);



> evalf(subs(Tj = 300, FInt));

$$1050.478057 \quad (52)$$

> Fint3000 := evalf(subs(Tj = 3000, FInt));

$$Fint3000 := 1051.513731 \quad (53)$$

> cp; rho;

$$880$$
$$1450 \quad (54)$$

> Qint := 0.96·29.86·cp·rho·Fint3000;

$$Qint := 3.846153909\ 10^{10} \quad (55)$$

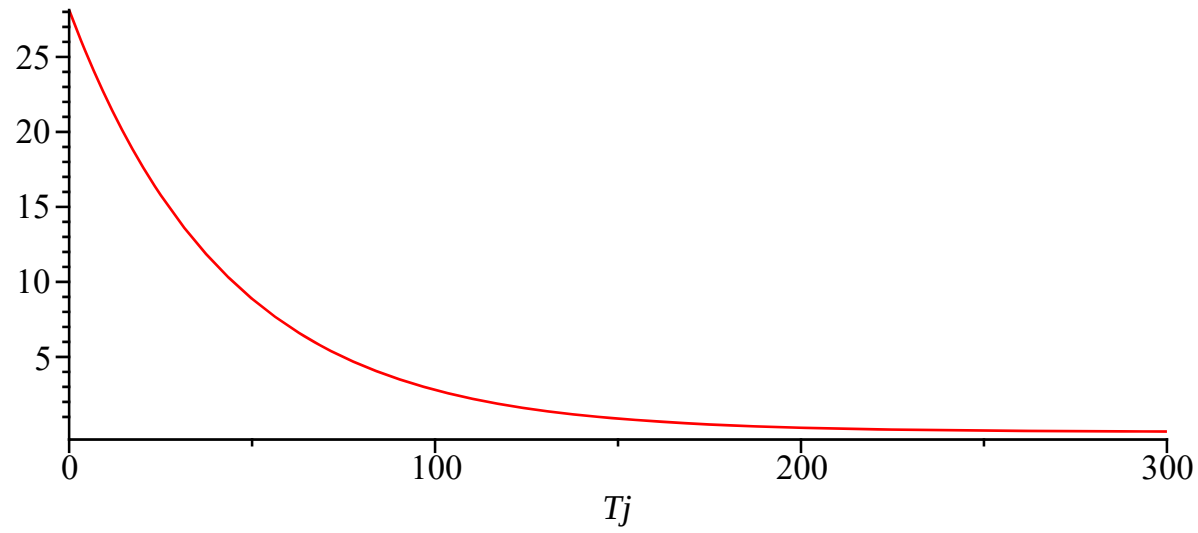
> q0

$$0.9811411587 \quad (56)$$

> Q := q0·0.96·29.86·exp(−b·t);

$$Q := 28.12500000\, e^{-0.02307649500\,Tj} \quad (57)$$

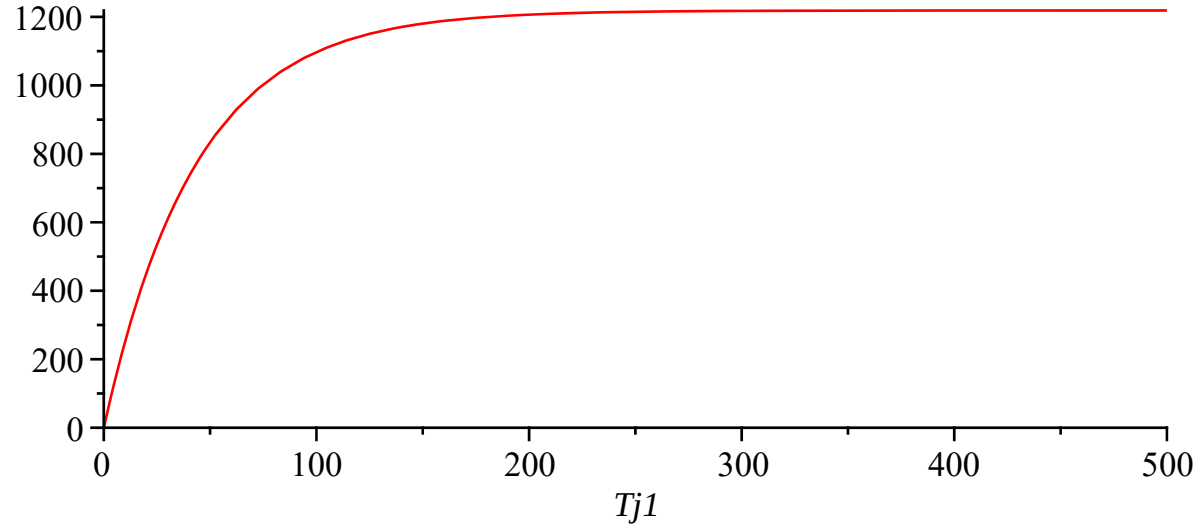
> plot(Q, Tj = 0..300);



> $Qint := int(Q, Tj = 0..Tj1); \# w \cdot a$

$$Qint := 1218.772608 - 1218.772608\, e^{-0.02307649500\, Tj1} \quad \textbf{(58)}$$

> $plot(Qint, Tj1 = 0..500);$



> $Qges := evalf(subs(Tj1 = 3000, Qint)) \cdot 365.25 \cdot 24 \cdot 3600; \# J$

$$Qges := 3.846153845\ 10^{10} \quad \textbf{(59)}$$

>

Mit dem größeren Abstand der Bohungen von 29.86 m ist die mittlere Wärmestromdichte nur noch etwa ein Drittel der früher für 11 m Abstand berechneten. Als Folge steigt die maximale Temperatur nur noch bis 17,3323 K über die Bergtemperatur. Das hat aber für die Berechnung der Temperatur um eine Linienquelle keine Auswirkungen. Die Temperatur direkt an der Behältergrenze ist genau so hoch wie vorher, nur der Einfluß der benachbarten Bohrungen ist nun geringer.